What is Applied Topology
(A role for topology in Neuroscience and Material science)

Think less about developing abstract

The blue brain project

- digital reconstruction of part of a rat
  microcircuitry of layer 2-6 of hand-limb
  somatosensory cortex of two-week-old rat.

- simulate activities

Reconstruction?

- layer heights, properties
- neuron morphologies and explicit pairwise connection
  probabilities between individual neurons

- biological organizing principles.

- 42 microcircuits of approx. 3.1 * 10^10 neurons
  8 million connections
  37 million synapses
- reproduced multiple in vitro and in vivo experiments &/or
  preclinical tests
Where does topology come in?

Topology of the micro-scale and its significance in emergent phenomena.

Why topology?

- beyond graph theory get "connectome"
- proximity & connectivity
- identifying local-to-global phenomena
- higher order connectivity

Topological tools:

Importance of direction

(a) undirected graphs — electrical synapses
(b) directed graphs — chemical synapses

So will use ordered simplicial complex: collection of

of fin. ordered sets \( s \in S \Rightarrow 2^s \cup 0 \)

if \( a \) has \( n+1 \) els it is a \( n \)-simplex
Example \[
\{ \varnothing, (1,2), (3), (1,2), (1,3), (2,3), (1,2,3) \}
\]

Non-example: \[ (1,2,3) \] but not \[ (1,3) \to (3,1) \].

Directed flag complexes

\[ S = (V, E, \tau : E \to V \times V) \] directed graph

directed flag complex associated to \( S \) is

abstract ordered set \( \text{spr} \) of \( S \) with

\[ \text{spr} = V \]

\[ \text{spr} = \{ (v_0, -v_n) \mid (v_i, v_j) \in E \ \forall \text{o.i.j.k} \} \]

i.e., directed \( (n+1) \)-graph in \( S \).
Analysis of reconstructed microcircuits

First show that these directed flag complexes are not random.

distribution of simplexes: count # of simplexes in the biological reconstruction and compare w/ various random models.

Compare w/ actual in vitro validation "patch clamping" which shows e.g. a lot of 2-cells "in nature"
Also calculated the homological dim of the complexes

All the rats have homological dim = 5
but have different Betti-numbers.

Looking at Euler Characteristic versus Betti-5
Can actually separate the rats.

Also compared with C. Elegans (a type of flat worm)
again get very complex directed flag complexes
i.e. high homological dim 12.

Interesting question of functional role of simplices? (graph)

How does topology of microcircuits influence its electrical activity,
(work in progress)
Other application?

Analyzing nanoporous materials, "Zeolites" are very well-known geometries that have important applications to gas capture and storage and catalysts.

Goal 1: use TDA to detect high-performance materials which minimize exposure.

Make point cloud data set of Zeolites and use persistent homology of resulting Victor-Rep complexes.

Make a similarity measure ("topological similarity") between materials.

Result: chemical behavior depends strongly on the topology.