$C^*$-algebras of Right-angled Artin Monoids

\[ H^1 = \ell^2(\mathbb{N}) \]

$S \in \Theta(H)$ given by

$S(\alpha_n) = \alpha_{n+1}

$C^*(S)$ C*-alg. generated by $S: H^1 \to H^1$

Universal properties of $C^*(S)$ (2003, 5.3, 5.6b)

Universal C*-alg. generated
by $\alpha$, $\beta$, $\gamma$

The compact $a \in C^*(S)$, we
get a state

$\theta \to \mathcal{K} \to \mathcal{L} \to C(\mathbb{N}) \to 0$

We can also do $S_0 \alpha_n = \alpha_{2n}$
$S_1 \alpha_n = \alpha_{2n+1}$

$C^*(S_0, S_1) = \mathcal{E}a = C^*<S_0, S_1>$

$s_0 s_0 = s_1 s_1 = 1, \\
\overline{s_0 s_1} = S_0 S_1 = 0$

Inside of $\mathcal{E}a$ we also have an ideal
that looks like the compact. We get in $SESS'$
Such a graph is called "co-irreducible." 

Irreducible graph: 1-path component.

Definition: (Euler characteristic) of \( G \)

\[ \chi(G) = \sum \frac{(-1)^k}{k} \text{ for } k \text{ p-simplex} \]

Degree of a simplex:
a p-simplex is a cycle with \( (p+1) \)-faces.

Definition (Cosp-Laca 02)

In a graph, \( C^\ast \)-algebra associated to the Anfin - Its manifold of \( G \) is

\[ C^\ast(A^n_G) = C^\ast \left[ \begin{array}{c} S \backslash S \cup S \cup v \cup V \end{array} \right] \]

Observation:

\[ C^\ast(A^{n+}) = C^\ast(A^{n}_{\text{co}}) \]

\( \otimes \cdots \otimes C^\ast(A^{n}_\text{co}) \)

When

\[ P = P_1 + P_2 \cdots + P_n \]

Partition of \( P \) into its co-irreducible components.
Examples:

\[ \gamma \]

... a (co)inductive

\[ \uparrow \]

But

\[ \text{What is } C^*(A^+ \mathbb{R})? \]

Question: When you look at free

directed graphs when
do we get the same C*-algebra?

Strategy 1: Classify \( C^*(\mathcal{F}A^+) \) using \( K\)-theory

Then: "Counts"

Thm: Cuntz, Ech. 12

For any \( n \),

\[ K_n (C^*(A^+) ) \cong \mathbb{Z} \oplus 0 \]

Proof: Brown Connes conjecture holds

For IM group Ap since it

has the Haagerup property.
For reducible 1

When \( q \) is co-indecomposable with

\[
1 < l_1 < \text{rank } X(q) \neq 0 \text{ in } G_r
\]

you have

\[
0 \to K \to C(A_r^+) \to \sigma_{\text{rank } X(q)} \to 0
\]

with \( n \)-fying

\[
\begin{array}{c}
\xrightarrow{\alpha(q)} \\
\xrightarrow{x(q)} \\
\xrightarrow{\sigma_{\text{rank } X(q)}}
\end{array}
\]

Why does \( X(q) \) come in?

When you compute \( K \to C^0(A_r^+) \)

in \( K \)-theory you have to

add and subtract stuff, why

much in the same way you

add and subtract steps to

get \( X(q) \).
Co-indecomposable \( 2 \).

\[ \mu_1 > 1, \quad \gamma(\mu) = 0 \]

We have:

\[ 0 \to K \to \gamma^* (A^*) \to 0_1 \to 0 \]

with \( K = H_G \)

\[ \begin{array}{c}
\mathbb{A} \\
\uparrow \\
\mathbb{H}
\end{array} \to \begin{array}{c}
\mathbb{H} \\
\uparrow \\
\mathbb{F}
\end{array} \to \begin{array}{c}
\mathbb{F}
\end{array} \]

\[ 0_1 \to \text{until Kirillov problem} \]

Problems may:

Stable, purely infinite

Until purely infinite

Stable, many AF / PI

Until, more AF / PI
Thm (\text{-Reshort\text{-Ruit})}

\[ 0 \rightarrow \mathbf{K} \rightarrow \mathbf{E} \rightarrow \mathbf{A} \rightarrow 0 \]

with \( \mathbf{E} \) a UCT-kirillov algebra on classified by their six-term exact sequence with inner

- \( K_-(\mathbf{A}) \) \& \( g_{\mathbf{E},A} \)

- \( K_1(\mathbf{A}) \) free.

- \( \text{rank } K_1(\mathbf{A}) \leq \text{rank } K_0(\mathbf{A}) \)

Thm (\text{-Lift-Luit})

with \( p \), \( p' \) are co-inducible with \( \Pi_{\mathbf{P}} \),

\[ |p'| | > 1 \text{ } \text{in } \text{h} \text{w} \text{r} \text{m} \]

\[ C^* (A^r_{p'}) = C^* (A^r_{p}) \Rightarrow \chi(p) = \chi(p') \]
Very little progress.

Theorem

\[ \text{Def: } \quad \psi(n) = \# \{ i \mid \pi_i > nx \} \]

\[ \text{Thm } (\psi - (i - \text{point}) \]

For a given graph \( P, p \)

where

\[ C(\Delta^*_{p}) \approx C(\Lambda^*_{p}) \]

Proof:

\[ \psi(n) = \psi(n') \]

\[ \text{1) } N_x(n) + N_{-x}(n) = N_x(n') \]

\[ \text{2) } N_x(n') \cup N_{-x}(n') \]

\[ \text{3) } N_x(n') \cup \text{or} \]

\[ \text{if } k > 0 \]

\[ \sum_{k > 0} N_k(n') = \sum_{k > 0} N_k(n') \approx \mu \]
Suppose \( \mathbf{R} \uparrow \mathbf{a} \mathbf{a} \) satisfies \( \exists \mathbf{x} \leq 1 \mathbf{s} : \mathbf{s} \leq \mathbf{i} - 1 \mathbf{s} \).

Q: Can we purchase \( \mathbf{s} \ldots \mathbf{u} \) to get \( \mathbf{t} \ldots \mathbf{u} \) satisfying the relation exactly? 

**Fact:** It is well known that \( \mathbf{R} \) and \( \mathbf{R'} \) are compatible.

But

\[
\mathbf{e} \mathbf{s} (\mathbf{t} \mathbf{e} \mathbf{r}) : \quad \mathbf{r} \times \mathbf{A} \text{ is only compatible when } \mathbf{A} \text{ is finite dimensional.}
\]

**Thm:** When \( \mathbf{t}(\mathbf{r}) = 0 \),

\[
\mathbf{c} \cdot (\mathbf{A}^+_{\mathbf{r}}) \text{ is compatible if}
\]

\[
\exists \mathbf{N} \in \mathbf{1} : \quad 2 \mathbf{k} \leq 1 \\
\mathbf{k} \neq 1 . \quad \square
\]

This is exactly the extent of our knowledge in this direction.