

Bachelor projects for mathematics and
mathematics-economics

Department of Mathematical Sciences
University of Copenhagen

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Introduction

This is a catalogue of projects suggested by the researchers at the Department of Mathematical Sciences for students in the B.S. programs in mathematics and mathematics-economics. It is important to note that such a catalogue will never exhaust all possibilities – indeed, if you are not finding what you are looking for you are strongly encouraged to ask the member of our staff you think is best qualified to help you on your way for suggestions of how to complement what this catalogue contains. Also, the mathematics-economics students are encouraged to study the searchable list of potential advisors at the Economy Department on

<http://web.econ.ku.dk/polit/studerende/Speciale/vejlederoversigt/>.

If you do not know what person to approach at the Department of Mathematical Sciences, you are welcome to try to ask

- the director of studies (Ernst Hansen, erhansen@math.ku.dk) or
- the associate chair for education (Tinne Hoff Kjeldsen, thk@math.ku.dk).

When you have found an advisor and agreed on a project, you must produce a contract (your advisor will know how this is done), which must then be approved by the director of studies at the latest during the first week of a block. The project must be handed in during the 7th week of the following block, and an oral defense will take place during the ninth week.

We wish you a succesful and engaging project period!

Best regards,

Tinne Hoff Kjeldsen
Associate chair

Ernst Hansen
Director of studies

1 Finance

1.1 Rolf Poulsen and David Skovmand

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Relevant interests:

Finance.

Suggested projects:

- OPTION PRICING [Fin1]
Pricing and hedging of exotic options (barrier, American (Longstaff & Schwartz (2000)), cliquet). A detailed investigation of convergence in of the binomial model (papers by Mark Joshi). Model calibration as an inverse problem (Derman & Kani (1995)). Incomplete models; equilibrium pricing (Carr & Madan (2000), Garleanu, Pedersen & Allen Poteshman (2009)).
- STOCHASTIC INTEREST RATES [Fin1]
Yield curve estimation. Estimation of dynamic short rate models. Calibration and the forward algorithm. Derivative pricing with applications to embedded options in mortgage products, the leveling algorithm. Risk management for mortgagors and pension funds.
- OPTIMAL PORTFOLIO CHOICE [Fin1]
The effects of parameter uncertainty on optimal portfolio choice (Kan & Zhou (2007) for instance). *Betting Against Beta* (Frazzini & Pedersen (2014)) and other CAPM-related stuff. Optimal multi-period investment with return predictability and transaction costs (Garleanu & Pedersen (2013)).
- STOCHASTIC VOLATILITY [Fin1 and preferably, but not necessarily, FinKont]
What is volatility? (There are at least a handful of definitions that all more or less are the same in the Black-Scholes model – but not in more general models.) How does volatility behave empirically? How does (stochastic) volatility affect (plain vanilla) option pricing? How can variance swaps (and other volatility derivatives) be priced and hedged?

2 Operations research

2.1 Trine K. Boomsma and Salvador Pineda

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Relevant interests:

Mathematical programming, stochastic programming, dynamic programming, real options, energy applications.

Suggested projects:

- MULTI-STAGE STOCHASTIC PROGRAMMING VS. STOCHASTIC DYNAMIC PROGRAMMING [OR1 + one of the following courses: Advanced mathematical programming, OK, GAMS, OR2, Stochastic Programming]

Stochastic programming is a mathematical framework that allows to solve optimization problems when some of the parameters involved are not fully known. For example, stochastic programming can be used to address both everyday issues such as “*which road should I take to get to work without knowing the traffic level in each of them?*” and complicated problems such as *determining the optimal portfolio of assets in a market without knowing how prices will evolve.*

However, the decisions made in the two previous examples are not final, but can be modified as uncertain information discloses. For example, if we decide to drive to work through road A and we realize after some kilometres that the traffic is very high, we may be able to switch to road B. Likewise, if we observe high prices of a given asset, we will try to increase the share of such asset to improve our final profit. Multi-stage stochastic programming succeeds to solve problems in which decisions may change as uncertain information becomes available. However, these type of problems present the curse of dimensionality, which basically means that the number of equations and variables drastically increase with the number of decision stages.

Alternatively, these problems can be solved using dynamic programming, which is a methodology that requires that decisions at each stage are discretized. For example, in the case of the assets, we should impose that assets can be sold or bought in lots of 10. By doing so, we can obtain the solution to the initial problem by solving smaller subproblems. While dynamic programming reduces the computational burden of some models, it does not provide the exact optimal solution because of the discretization of the decision variables.

Within this framework, this project will focus on comparing these two mathematical tools and determine in which cases multi-stage stochastic programming overcomes dynamic programming and vice versa. To evaluate each methodology in a real-life problem, we propose an energy planning model that aims at determining the electricity production of different types of generating units facing the uncertainty of electricity demand level and wind power production.

2.2 Giovanni Pantuso

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Relevant interests:

Operations Research, Stochastic Programming, Integer Programming, Decision Making

Suggested projects:

- FOOTBALL TEAM COMPOSITION [OR1, one between OR2, GAMS and Modeling, and Advanced OR: Stochastic Programming]
Selecting football players is one of the most crucial decisions for a football club. When composing a team of players clubs must take into account a number of issues such as the regulations of the competitions where the team participates, the requirements from the coach, budget limits and contracts duration. In this project you will write, implement, and test a mathematical model for the football team composition problem.
- DEMAND-BASED DISCRETE OPTIMIZATION [OR1, one between OR2, GAMS and Modeling, and Advanced OR: Stochastic Programming]
In this project you will study optimization models which integrate customers behavior. Such models would allow a wide variety of service providers to explicitly take into account customers preferences when deciding the services to offer and their prices. As an example, the manager of a movie theater could use such model to decide which movies to show in the theater rooms as well as the pricing for individual/categories of customers. You will initially describe customers choice models available in the literature, such as logit models. Then you will describe an optimization model which integrates customers choices, implement simple instances in e.g., GAMS or Cplex, and discuss the results.
- CAR-SHARING PLANNING PROBLEMS [OR1, one between OR2, GAMS and Modeling, and Advanced OR: Stochastic Programming]
Car-sharing services are recently emerging in many cities as an alternative to classic public transportation means. Such services generate new and complex planning problems. In this project you will study, describe, and model a planning problem faced in car-sharing services and possibly implement and test the model in e.g., GAMS or Cplex.

3 Algebra and number theory

3.1 Henrik Holm

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Relevant interests:

Rings, modules, homological algebra, category theory.

The prerequisites for the following projects are the courses [Alg 1] and [Alg 2].

Details, and possibly additional suggestions for projects, may be found at my homepage <http://www.math.ku.dk/~holm/>

Suggested projects:

- COMPLETION OF RINGS [Alg1, Alg2]

Given an ideal I in a commutative ring R one can construct the I -adic completion \widehat{R}_I . For example, $\widehat{k[x]}_{(x)}$ is the formal power series ring and $\widehat{\mathbb{Z}}_{(p)}$ is the ring of p -adic integers. The aim of this project is to define adic completions and to investigate their basic properties.

Literature: H. Matsumura, "Commutative ring theory".

- GROUP (CO)HOMOLOGY [Alg1, Alg2]

To a group G one can associate a sequence of (abelian) homology groups $H_n(G)$ and cohomology groups $H^n(G)$ that contain information about G . For example, $H_1(G) = G_{\text{ab}}$ is the abelianization of G . The aim of this project is to define group (co)homology and to give group theoretical descriptions of the lower (co)homology groups.

Literature: P. J. Hilton and U. Stambach, "A course in homological algebra".

- GRÖBNER BASES [Alg1, Alg2]

Given an ideal I in the polynomial ring $k[x_1, \dots, x_n]$ and a term ordering \leq one can always find a so-called Gröbner basis g_1, \dots, g_m of I with respect to \leq . For example, a Gröbner basis for the ideal $I = (y^2 - x^3 + x, y^3 - x^2)$ with respect to the lexicographic ordering (where $x \geq y$) consists of $g_1 = y^9 - 2y^6 - y^4 + y^3$ and $g_2 = x - y^7 + y^4 + y^2$. Gröbner bases are powerful tools to solve e.g. polynomial equations and the ideal membership problem. The aim of this project is to define, and to show the existence of, Gröbner bases, and to demonstrate some applications.

Literature: N. Lauritzen, "Concrete abstract algebra" and D. Cox, J. Little, and D. O'Shea, "Ideals, varieties, and algorithms".

- INJECTIVE MODULES [Alg1, Alg2]

An object in a category is called injective if it has a certain lifting property. For example, the injective objects in the category of abelian groups are precisely the divisible abelian groups (such as the group of rational numbers \mathbb{Q} and the Prüfer groups $\mathbb{Z}(p^\infty)$ where p is a prime). The aim of this project is to develop the theory of injective modules over an arbitrary ring.

Literature: F. W. Anderson and K. R. Fuller, “Rings and categories of modules” and E. E. Enochs and O. M. G. Jenda, “Relative homological algebra”.

- THE LOWER K-GROUPS OF A RING [Alg1, Alg2]

The algebraic K-theory of a ring R is a certain sequence $K_n(R)$ of abelian groups that contains information about R . For example, if R is a field, then $K_0(R) = \mathbb{Z}$ is the additive group of integers and $K_1(R) = R^\times$ is the multiplicative group of units in R . The aim of this project is to define and investigate the lower K-groups for certain classes of rings.

Literature: J. Rosenberg, “Algebraic K-theory and its applications”.

- ADJOINT FUNCTORS IN CATEGORY THEORY [Alg1, Alg2]

Adjoint functors are important and abundant in mathematics. For example, the forgetful functor $U: \mathbf{Vect} \rightarrow \mathbf{Set}$ from the category of (real) vector spaces to the category of sets has a left adjoint $V: \mathbf{Set} \rightarrow \mathbf{Vect}$, which to each set X associates the (real) vector space with basis X . The aim of this project is to develop the basic theory of adjoint functors and to prove Freyd’s Adjoint Functor Theorem and The Special Adjoint Functor Theorem.

Literature: S. Mac Lane, “Categories for the working mathematician”.

3.2 Christian U. Jensen

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Relevant interests:

Galois theory. Algebraic number theory.

Suggested projects:

- INTRODUCTORY GALOIS THEORY [Alg2]

This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics, but is also a very rich field that can be studied in its own right.

- INTRODUCTION TO ALGEBRAIC NUMBER THEORY [Alg2]

Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

3.3 Ian Kiming and Fabien Mehdi Pazuki

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Relevant interests:

Algebraic number theory and arithmetic geometry.

Suggested projects:

- INTRODUCTION TO ALGEBRAIC NUMBER THEORY [Alg2]
Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.
- FIRST CASE OF FERMAT'S LAST THEOREM FOR REGULAR EXPONENTS [Alg2]
The project studies the proof of Fermat's last theorem for 'regular' prime exponents p in the so-called first case: this is the statement that $x^p + y^p + z^p = 0$ does not have any solutions in integers x, y, z not divisible by p . The project involves studying some introductory algebraic number theory which will then also reveal the definition of 'regular primes'.
- p -ADIC NUMBERS [Alg2]
The real numbers arise from the rational numbers by a process called 'completion'. It turns out that the rational numbers (and more generally any algebraic number field) has infinitely many other 'completions', namely one associated to each prime number p . The fields that arise in this way are called the fields of p -adic numbers. They have a lot of applications in many branches of mathematics, not least in the theory of Diophantine equations, i.e., the question of solving in integers polynomial equations with integral coefficients.
- HASSE-MINKOWSKI'S THEOREM ON RATIONAL QUADRATIC FORMS [Alg2]
A rational quadratic form is a homogeneous polynomial with rational coefficients. The Hasse-Minkowski theorem states that such a polynomial has a non-trivial rational zero if and only if it has a non-trivial zero in the real numbers and in all fields of p -adic numbers. The latter condition can be translated into a finite number of congruence conditions modulo certain prime powers and thus one obtains an effective criterion. The project involves an initial study of p -adic numbers.
- CONTINUED FRACTIONS AND PELL'S EQUATION [Alg2]
The project studies the theory of continued fractions and how this can be applied to determining units in quadratic number rings. This has applications to the

study of Pell (and ‘non-Pell’) equations, i.e., solving equations $x^2 - Dy^2 = \pm 1$ in integers for a given positive, squarefree integer D .

- CLASS GROUPS OF QUADRATIC NUMBER FIELDS AND BINARY QUADRATIC FORMS [Alg2]

A quadratic number field is a field obtained from \mathbb{Q} by adjoining a number of form \sqrt{D} where D is an integer that is not a square (in \mathbb{Z} .) The class group attached to such a field measures how far its so-called ring of integers is from being a unique factorization domain. These class groups are necessary to study of one wants to understand integer solutions to equations of form $ax^2 + by^2 = c$ for given integers a, b, c .

- MODULAR FORMS ON $SL_2(\mathbb{Z})$ [Alg2, KomAn]

This project studies modular forms on $SL_2(\mathbb{Z})$. These are initially analytic objects and thus a certain, minimal background in complex analysis is required. Modular forms turn out to have a lot of deep connections to arithmetic, and one can use this project as a platform for a later study of the more general modular forms on congruence subgroups of $SL_2(\mathbb{Z})$. These are very important in modern number theory and are for instance central in Andrew Wiles’ proof of Fermat’s last theorem.

- INTRODUCTORY GALOIS THEORY [Alg2]

This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics.

- GROUP COHOMOLOGY [Alg2]

Group cohomology is a basic and enormously important mathematical theory with applications in algebra, topology, and number theory. The project will study the initial theory starting with cohomology of discrete groups and then perhaps move on to cohomology of profinite groups. This project can be used as a platform for continuing with study of Galois cohomology and Selmer groups.

- THE THEOREM OF BILLING–MAHLER [Alg2, EllKurv]

A big theorem of Barry Mazur (1977) implies in particular that if n is the order of a rational point of finite order on an elliptic curve defined over \mathbb{Q} then either $1 \leq n \leq 10$ or $n = 12$. Thus, in particular, $n = 11$ is impossible. This latter statement is the theorem of Billing and Mahler (1940). The project studies the proof of the theorem of Billing–Mahler which will involve a bit more theory of elliptic curves as well as an initial study of algebraic number theory. The impossibility of $n = 13$ can also be proved with these methods.

- TORSION POINTS ON ELLIPTIC CURVES [Alg2, EllKurv]

The project continues the study of elliptic curves defined over \mathbb{Q} in the direction of a deeper study of (rational) torsion points. There are several possibilities here, for instance, parametrizations of curves with a point of a given, low order,

generalizations of the Nagell–Lutz theorem, the structure of the group of torsion points on elliptic curves defined over a p -adic field (Lutz’ theorem).

- **PRIMALITY TESTING [Alg2]**
How can one decide efficiently whether a large number is a prime number? The project will study one or more of the mathematically sophisticated methods of doing this: the Miller–Rabin probabilistic primality test and/or the more recent Agrawal–Kayak–Saxena deterministic primality test. The project will include an initial study of algorithmic complexity theory.
- **FACTORIZATION ALGORITHMS [Alg2]**
How can one find the prime factorization of a large number? The project will study one or more of the mathematically sophisticated methods of doing this: the Dixon factorization method, factorization via continued fractions, the quadratic sieve. The project will include an initial study of algorithmic complexity theory.
- **OPEN PROJECT [?]**
If you have some ideas on your own for a project within the general area of number theory, you can always come and discuss the possibilities with me.

Previous projects:

- **THE AGRAWAL-KAYAK-SAXENA PRIMALITY TEST [Alg2]**
- **SELMER GROUPS AND MORDELL’S THEOREM [Alg3, EllKurv]**
- **HASSE–MINKOWSKI’S THEOREM ON RATIONAL QUADRATIC FORMS [Alg2]**
- **TORSION POINTS ON ELLIPTIC CURVES [Alg2, EllKurv]**
- **FACTORIZATION VIA CONTINUED FRACTIONS [Alg2, Krypto]**
- **THE POHLIG-HELLMAN ALGORITHM FOR COMPUTING DISCRETE LOGARITHMS [Alg2]**
- **SCHOOF’S ALGORITHM [Alg3, EllKurv]**

3.4 Other projects

Other projects in this area can be found with

- Jesper Grodal (7.1)
- Morten S. Risager (4.4)

4 Analysis

4.1 Bergfinnur Durhuus

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Relevant interests:

Analysis: Operator theory, differential equations. Mathematical physics: Quantum mechanics, statistical mechanics. Discrete mathematics: Graph theory, analytic combinatorics, complexity theory,

Suggested projects:

- GRAPH COLOURING PROBLEMS [Dis1, An1]
Problems originating from various areas of mathematics can frequently be formulated as colouring problems for certain types of graphs. The four-colour problem is probably the best known of colouring problems but there is a variety of other interesting colouring problems to attack
- COMBINATORICS OF GRAPHS [Dis1, An1, ComAn]
Counting of graphs specified by certain properties (e.g. trees) is one of the classical combinatorial problems in graph theory having applications in e.g. complexity theory. The method of generating functions is a particularly effective method for a large class of such problems making use of basic results from complex analysis
- UNBOUNDED OPERATORS AND SELF-ADJOINTNESS [An2]
Many of the interesting operators playing a role in mathematical physics, in particular differential operators of use in classical and quantum mechanics, are unbounded. The extension of fundamental results valid for bounded operators on a Hilbert space, such as the notion of adjoint operator and diagonalisation properties, is therefore of importance and turns out to be non-trivial

Previous projects:

- CLIFFORD ALGEBRAS, SPIN GROUPS AND DIRAC OPERATORS [Alg1, An2]
- RAMSEY THEORY [Dis1, An1]
- CAUSAL STRUCTURES [An1, Geom2]
- THE TUTTE POLYNOMIAL [Dis1, An1]
- KNOT THEORY AND STATISTICAL MECHANICS [Dis1, AN1]
- GRAPH 3-COLOURINGS [Dis1, An1]
- MINIMAL SURFACES [Geom1, An1]
- PLANAR GRAPHS [Dis1, AN1]

4.2 Jan Philip Solovej

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Relevant interests:

Mathematical Physics, Quantum Mechanics, Spectral Theory, Partial Differential Equations

Suggested projects:

- PARTIAL DIFFERENTIAL EQUATIONS: THE WAVE EQUATION AND THE HEAT EQUATION [An1,An2]

The goal of the project is to derive the solution formulas for the initial value problems for the wave equation (homogeneous and inhomogeneous) and the heat equation. Moreover, the project will focus on the strong and weak Huygens principles for the wave equation and on the maximum principle for the heat equation.

Literature: F. John, Partial Differential Equations or L.C. Evans Partial Differential equations.

- FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS [An1,An2]

The project considers first order partial differential equations, in particular, quasi-linear equations. The goal is to describe the method of characteristics and to study equations that exhibit shock formation.

Literature: F. John, Partial Differential Equations

4.3 Henrik L. Pedersen

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Relevant interests:

Complex analysis. Special functions. Orthogonal polynomials and moment problems.

Suggested projects:

- THE GAMMA FUNCTION [An1,KomAn]

Euler's Gamma function is the most important of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.

- ENTIRE FUNCTIONS [An1, Koman]

Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.

- **BOUNDARY BEHAVIOUR OF POWER SERIES** [KomAn]
A power series converges inside the disk of convergence, and diverges outside the circle. What happens on the boundary? The sum of the geometric series has a holomorphic extension to the entire complex plane except 1. If we remove a lot of terms from this series it turns out that the sum function has the unit circle as a natural boundary, meaning that it cannot be extended holomorphically to any arc of that circle. What is going on?
- **SUBHARMONIC FUNCTIONS** [KomAn, Measure theory]
The real part u of a holomorphic function is harmonic, meaning that its Laplacian is zero: $\Delta u = \partial_{xx}^2 u + \partial_{yy}^2 u \equiv 0$. A subharmonic function u in the complex plane satisfies $\Delta u \geq 0$. For these functions versions of the maximum principle and of Liouville's theorem hold
- **PICARD'S THEOREMS** [KomAn, some measure theory]
If you are presented with an entire function f and you have two different complex numbers not in the image set $f(\mathbb{C})$ then f is constant. This result is known as Picard's little theorem.
- **RIEMANN'S MAPPING THEOREM** [KomAn]
Any simply connected region \mathcal{D} in the complex plane except the plane itself is conformally equivalent to the open unit disk Δ , meaning that there exists a holomorphic and bijective mapping $\varphi : \Delta \rightarrow \mathcal{D}$.
- **MÜNTZ-SZASZ' THEOREM** [KomAn, Functional Analysis]
Let $\{\lambda_k\}$ be an increasing sequence of positive numbers. When is the span of the power-functions $\{1, x^{\lambda_1}, x^{\lambda_2}, \dots\}$ dense in the space of continuous functions on $[0, 1]$? Answer: exactly when $\sum_{k=1}^{\infty} 1/\lambda_k = \infty$!
- **PALEY-WIENER'S THEOREM** [KomAn, some measure theory]
The Fourier transform $\widehat{\phi}$ of a function ϕ from the Hilbert space $L_2(-a, a)$ can be extended to an entire function of exponential type (meaning that its growth is dominated by $e^{K|z|}$ for all large $|z|$). Conversely, any entire function of exponential type is in fact the Fourier transform of an L_2 -function of a finite interval.

4.4 Morten S. Risager

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Relevant interests:

Number theory, automorphic forms, complex analysis, Riemann surfaces.

Suggested projects:

- **THE PRIME NUMBER THEOREM** [KomAn, An2]
The prime number theorem gives a quantitative version of Euclid's theorem about the infinitude of primes: it describes how the primes are distributed among the integers. It was conjectured 100 years before the first proof.

- TWIN PRIMES AND SIEVE THEOREMS [KomAn, An2]
Very little is known about the number of twin primes. Using sieve methods one can show that the sum of reciprocals of twin primes is convergent. Still it is not known if there are only finitely many or not.
- THE FUNCTIONAL EQUATION FOR RIEMANN'S ZETA FUNCTION [KomAn, An2]
Using methods from Fourier analysis - in particular Poisson summation - one investigates the properties of Riemann's famous zeta function.

Previous projects:

- ELEMENTARY METHODS IN NUMBER THEORY, AND A THEOREM OF TERENCE TAO. [An2, ElmTal]
- PRIMES IN ARITHMETIC PROGRESSIONS [KomAn, An2]
- SMALL EIGENVALUES OF THE AUTOMORPHIC LAPLACIAN AND RADEMACHERS CONJECTURE FOR CONGRUENCE GROUPS [KomAn, An3]
- WEYLS LAW AND THE GAUSS' CIRCLE PROBLEM [An2]
- THE BOMBIERI-VINOGRADOV THEOREM [AnTal]
- PRIMES CLOSE TOGETHER [AnTal]
- DIRICHLETS UNIT THEOREM [Alg 2]
- P-ADIC NUMBERS AND THE HASSE-MINKOVSKI THEOREM [Alg2]
- HIGHER ORDER RECIPROCITY [Elmtal, Alg2]
- THE AGRAWAL-KAYAK-SAXENA ALGORITHM FOR PRIMALITY TESTING [Alg2]
- PELL'S EQUATION AND ARCHIMEDES' REVENGE [ElmTal, Alg2]
- THE PHRAGMÉN LINDELÖF PRINCIPLE [KomAn]
- MODULAR FORMS [KomAn]

4.5 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- THE HEISENBERG GROUP [An1,An2]
The Heisenberg group is important, for example because it is generated by the position and momentum operators in quantum mechanics. The purpose of this project is to study its representation theory. A famous theorem of Stone and von Neumann relates all irreducible representations to the Schrödinger representation acting on $L^2(\mathbf{R}^n)$.
- UNCERTAINTY PRINCIPLES [An1,Sand1,KomAn]
Various mathematical formulations of the Heisenberg uncertainty principle are studied. Expressed mathematically, the principle asserts that a non-zero function f on \mathbf{R} and its Fourier transform \hat{f} cannot be simultaneously concentrated. A precise version, called the Heisenberg inequality, expresses this in terms of standard deviations. A variant of the theorem, due to Hardy, states that f and \hat{f} cannot both decay more rapidly than a Gaussian function.
- THE PETER-WEYL THEOREM [An1,An2,Sand1]
The purpose of this project is to study $L^2(G)$ for a compact group G , equipped with Haar measure. The theorem of Peter and Weyl describes how this space can be orthogonally decomposed into finite dimensional subspaces, which are invariant under left and right displacements by G . Existence of Haar measure can be proved or assumed.

4.6 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- SOLITONS IN KORTEWEG-DE VRIES' EQUATION [An1]
Waves normally either spread out or break with time. However, the surprising emergence of special “solitary waves” (a.k.a. “solitons”) which retain a fixed profile with time can occur. Historically, this was first observed by Scottish naval engineer John Scott Russell in 1834, in a shallow water canal: This “translating wave” of water, as he named it, was so stable in shape that he could follow it by horseback long enough to determine its speed to be approximately 8 miles/hour. The KdV equation models this behavior and has many interesting features that you can investigate, such as the collision of several solitons.
- CURVE SHORTENING FLOW AND ISOPERIMETRY [An1,An2,Geom1]
The mean curvature flow for planar curves is used for proving the isoperimetric inequality: Of all the simple closed planar curves enclosing the same area, the

round circle has the least length. It is actually essential for the proof that the circle is a geometric soliton, see the KdV project. (Especially the non-convex case is very hard, so do not expect to finish the full proof).

- BROWNIAN MOTION AND THE HEAT EQUATION [An1, Sand1]
One can start with the random walk as a simple model for a diffusion process and derive the heat equation (and understand Einstein's proof from one of his three famous 1905 papers). There are extensions to modern versions involving fractional differentiation operators. I.e. how do you differentiate "half a time"?

5 Geometry

5.1 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- GLOBAL PROPERTIES OF CURVES (AND/OR SURFACES) [Geom1,An1]
The differential geometry studied in Geometry 1 is of a local nature. The curvature of a curve in a point, for example, describes a property of the curve just in the vicinity of that point. In this project the focus is on *global* aspects of closed curves, as for example expressed in Fenchel's theorem, which gives a lower bound for the total integral of the curvature, in terms of the perimeter.
- GEODESIC DISTANCE [Geom1,An1]
The geodesic distance between two points on a surface is the shortest length of a geodesic joining them. It turns the surface into a metric space. The project consists of describing some properties of the metric. For example Bonnet's theorem: *If the Gaussian curvature is everywhere ≥ 1 , then all distances are $\leq \pi$.*

5.2 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- MATHEMATICAL SOAP BUBBLES: MINIMAL SURFACES [An1]
There is a rich theory for surfaces of (zero or positive) constant mean curvature, and many interesting examples to visualize. Important aspects of existence and uniqueness can be studied (the difficulty of the project accordingly adjusted).
- VERY LONG GEODESICS: LIBERMAN'S THEOREM [An1, Geom1]
The theorem states that on a closed 2-dimensional surface with everywhere positive Gauß curvature, geodesic curve segments cannot be arbitrarily long and still remain injective: They must eventually self-intersect. One can also prove an upper bound for the length.

- THE UNIFORMIZATION THEOREM FOR SURFACES [An1,An2,Geom1/KomAn]
A special case of this classical theorem states that: Every closed 2-dimensional surface has a Riemannian metric of constant Gauß curvature. (This is a long and complicated proof, so do not expect your thesis to contain it in all details).

5.3 Meritxell Saez

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Relevant interests:

Advanced linear algebra, geometry, applied mathematics.

Suggested projects:

- CONICS IN PROJECTIVE GEOMETRY [LinAlgMat, Geom1]
Ellipses, parabolas and hyperbolas are all instances of the same object when considered in the projective plane. The project consists on first an introduction to projective geometry as a generalization of affine geometry and second the construction of families of conics with GeoGebra (www.geogebra.org).
- PERSPECTIVE IN PROJECTIVE GEOMETRY [LinAlgMat, Geom1]
The perspective one can see in a picture can be translated into a mathematical object by means of projective geometry. The project consists on first an introduction to projective geometry as a generalization of affine geometry and second the visualization of the perspective used both in art and computer vision.

5.4 Hans Plesner Jakobsen

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Relevant interests:

Unitaritet, Liegrupper, Liealgebraer, kvantiserede matrixalgebraer, kovariante differentialoperatorer i matematisk fysik, kvantiserede indhyldningsalgebraer

Suggested projects:

- SYMMETRIER [ca. 1 års matematik]
 - Diskrete symmetrier: Tapetgrupper, Krystallografiske grupper.
 - Kontinuerte symmetrier: Rotationsgruppen, Lorentzgruppen, PoincarÉ-gruppen, den konforme gruppe,...
- GAUSS-BONNET [Forudsætter ca. 1 års matematik]
Den måske mest fundamentale sætning i Euklidisk geometri er Thales' sætning, der siger, at summen af vinklerne i en trekant er 180° . Denne sætning kan

generaliseres til trekantede påglatte flader i rummet. Herved fremkommer Gauss-Bonnet's sætning i lokal udgave. Den globale udgave af sætningen leder til en invariant for kompakte flader: Eulerkarakteristikken.

- DE KANONISKE KOMMUTATORRELATIONER [ca. 1 års matematik + Hilbertrum]
Operatorerne Q og P givet ved $(QF)(x) = xF(x)$ og $(PF)(x) = -i(\frac{dF}{dx})(x)$ er forbundet via Fouriertransformationen, men kan Fouriertransformationen 'konstrueres' ud fra disse? Kan man bygge en bølgeoperator eller en Diracoperator ud af den harmoniske oscillator?
- LIEALGEBRAER [ca. 1 års matematik + (kan aftales)]
(F.eks.) Klassifikation. Dynkin diagrammer, Kac-Moody algebraer, super Liealgebraer. Hvad fik Borchers (bl.a.) Fieldsmedaljen for?
- MATRIX LIEGRUPPER [ca. 1 års matematik]
Bl.a. eksponentialfunktionen for matricer, tensorprodukter, duale vektorrum. Er der en forbindelse mellem Peter Weyl Sætningen og Stone Weierstrass Sætningen?

5.5 Other projects

Other projects in this area can be found with

- Nathalie Wahl (7.3)

6 Noncommutativity

6.1 Søren Eilers

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Relevant interests:

Advanced linear algebra related to operator algebras. Dynamical systems. Mathematics in computer science; computer science in mathematics.

Suggested projects:

- PERRON-FROBENIUS THEORY WITH APPLICATIONS [LinAlg, An1]
Methods involving matrix algebra lead to applications such as Google's PageRank and to the ranking of American football teams.
- DATA STORAGE WITH SYMBOLIC DYNAMICS [An1, Dis1]
Engineering constraints necessitate a recoding of arbitrary binary sequences into sequences meeting certain constraints such as “between two consecutive ones are at least 1, and at most 3, zeroes”. Understanding how this is done requires a combination of analysis and discrete mathematics involving notions such as entropy and encoder graphs.
- EXPERIMENTAL MATHEMATICS [LinAlg]
Design an experiment in Maple to investigate a mathematical problem, cf. www.math.ku.dk/~eilers/xm.

Previous projects:

- AN EXPERIMENTAL APPROACH TO FLOW EQUIVALENCE [An1]
- VISUALIZATION OF NON-EUCLIDEAN GEOMETRY [MatM, Geom1]
- PLANAR GEOMETRY IN HIGH SCHOOL MATHEMATICS [MatM]
- LIAPOUNOV'S THEOREM [MI]

6.2 Niels Grønbaek

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Relevant interests:

Banachrum, banachalgebra, kohomologi, matematikkens didaktik

Suggested projects:

- ET UNDERVISNINGSFORLØB PÅ GYMNASIALT NIVEAU [LinAlg, An1, Alg1, Geo1]
Projektet går ud på at tilrettelægge, udføre og evaluere et undervisningsforløb af ca. 2 ugers varighed i en gymnasieklasse.
- AMENABLE BANACH ALGEBRAS [An3]
Amenability of Banach algebras is an important concept which originates in harmonic analysis of locally compact groups. In the project you will establish this connection and apply it to specific Banach algebras such as the Banach algebra of compact operators on a Hilbert space.

6.3 Magdalena Musat

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Relevant interests:

Banach Spaces, Functional Analysis, Operator Algebras, Probability Theory

Suggested projects:

- GEOMETRY OF BANACH SPACES [Analysis 3]
A number of very interesting problems concerning the geometry of Banach spaces can be addressed in a bachelor project. For example, does every infinite dimensional Banach space contain an infinite dimensional reflexive subspace or an isomorphic copy of l_1 or c_0 ? Or, does there exist a reflexive Banach space in which neither an l_p -space, nor a c_0 -space can embed? Another project could explore the theory of type and cotype, which provides a scale for measuring how close a given Banach space is to being a Hilbert space.
- CONVEXITY IN BANACH SPACES [Analysis 3]
The question of differentiability of the norm of a given Banach space is closely related to certain convexity properties of it, such as uniform convexity, smoothness and uniform smoothness. This project will explore these connections, and study further properties of uniformly convex (respectively, uniformly smooth) spaces. The Lebesgue spaces L_p ($1 < p < \infty$) are both uniformly convex and uniformly smooth.
- HAAR MEASURE [MI]
This project is devoted to the proof of existence and uniqueness of left (respectively, right) Haar measure on a locally compact topological group G . For example, Lebesgue measure is a (left and right) Haar measure on \mathbb{R} , and counting measure is a (left and right) Haar measure on the integers (or any group with the discrete topology).

- FERNIQUE'S THEOREM [SAND 1, Analysis 3]

This project deals with probability theory concepts in the setting of Banach spaces, that is, random variables taking values in a (possibly infinite dimensional) Banach space. Fernique's theorem generalizes the result that gaussian distributions on \mathbb{R} have exponential tails to the (infinite dimensional) setting of gaussian measures on arbitrary Banach spaces.

6.4 Ryszard Nest

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Relevant interests:

Non-Commutative Geometry, Deformation Theory, Poisson Geometry

Suggested projects:

- CLIFFORD ALGEBRAS [LinAlg, Geom 1]

Clifford algebra is a family $C^{p,q}$ of finite dimensional algebras associated to non-degenerate bilinear forms which play very important role in both topology and geometry. The simplest examples are \mathbb{R} , \mathbb{C} and the quaternion algebra \mathbb{H} . The main result is the periodicity modulo eight of $C^{p,q}$, which has far reaching consequences (e.g., Bott periodicity, construction of Dirac operators) in various areas of mathematics.

- AXIOM OF CHOICE AND THE BANACH-TARSKI PARADOX [LinAlg, Analysis 1]

The axiom of choice, stating that for every set of mutually disjoint nonempty sets there exists a set that has exactly one member common with each of these sets, is one of the more "obvious" assumptions of set theory, but has far reaching consequences. Most of modern mathematics is based on its more or less tacit assumption. The goal of this project is to study equivalent formulations of the axiom of choice and some of its more exotic consequences, like the *Banach-Tarski paradox*, which says that one can decompose a solid ball of radius one into five pieces, and then rearrange those into two solid balls, both with radius one.

- FORMAL DEFORMATIONS OF \mathbb{R}^{2n} [LinAlg, Geom 1]

The uncertainty principle in quantum mechanics says that the coordinate and momentum variables satisfy the relation $[p, x] = \hbar$, where \hbar is the Planck constant. This particular project is about constructing associative products in $C^\infty(\mathbb{R}^{2n})[[\hbar]]$ satisfying this relation and studying their properties.

6.5 Mikael Rørdam

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Relevant interests:

Operator Algebras, Topics in Measure Theory, Discrete Mathematics

Suggested projects:

- TOPICS IN C^* -ALGEBRAS [Analysis 3]
 C^* -algebras can be defined either abstractly, as a Banach algebra with an involution, or concretely, as subalgebras of the algebra of bounded operators on a Hilbert space. They can be viewed as non-commutative analogues of spaces, since every commutative C^* -algebra is equal to the set of continuous functions on a locally compact Hausdorff space. Several topics concerning C^* -algebras and concerning the study of specific examples of C^* -algebras, can serve as interesting topics for a bachelor project.
- TOPICS IN MEASURE THEORY [MI]
We can here look at more advanced topics from measure theory, that are not covered in MI, such as existence (and uniqueness) of Lebesgue measure, or more generally of Haar measure on locally compact groups. Results on non-measurability are intriguing, perhaps most spectacularly seen in the Banach-Tarski paradox that gives a recipe for making two solid balls of radius one out of a single solid ball of radius one!
- TOPICS IN DISCRETE MATHEMATICS [Dis2 & Graf]
One can for example study theorems about coloring of graphs. One can even combine graph theory and functional analysis and study C^* -algebras arising from graphs and the interplay between the two (in which case more prerequisites are needed).

Previous projects:

- IRRATIONAL AND RATIONAL ROTATION C^* -ALGEBRAS [Analyse 3]
- CONVEXITY IN FUNCTIONAL ANALYSIS [Analyse 3]
- THE BANACH-TARSKI PARADOX [MI recommended]

6.6 Thomas Vils Petersen

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Relevant interests:

Functional analysis, analysis, Banach algebras of functions.

Suggested projects:

- CONVOLUTION ALGEBRAS [An2, MI]
These are Banach algebras of functions, and the product is the convolution product. Some possible topics:
 - Derivations on $L^1[0, 1]$.
 - Homomorphisms between weighted convolution algebras $L^1(\omega)$ on the half-line \mathbb{R}^+ .

6.7 Valerio Proietti

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Relevant interests:

Functional analysis, operator algebras, noncommutative geometry.

Suggested projects:

- NON-STANDARD ANALYSIS AND INVARIANT SUBSPACES [Logic, An2]
Non-standard analysis reformulates ordinary calculus using a logically rigorous notion of infinitesimal numbers. Abraham Robinson and Allen Bernstein used non-standard analysis to prove that every polynomially compact linear operator on a Hilbert space has an invariant subspace. Upon reading a preprint of this paper, Paul Halmos reinterpreted their proof using standard techniques.
Literature: P. Halmos, Invariant subspaces for Polynomially Compact Operators, Pacific Journal of Mathematics, 16:3 (1966) 433-437.
- COMPARING THEORIES OF INFINITESIMALS [Logic, Functional Analysis]
In modern calculus, historically relevant concepts such as "fluxions" or "infinitesimal numbers" are formally dealt with using epsilon-delta procedures. Non-standard analysis instead reformulates the calculus using a logically rigorous notion of infinitesimal numbers. In his 1995 article "Noncommutative geometry and reality", Connes develops a calculus of infinitesimals based on operators in Hilbert space. He proceeds to "explain why the formalism of non-standard analysis is inadequate" for his purposes. Katz & Katz analyze Connes' criticisms of non-standard analysis, and challenge some of his specific claims.
Literature: Katz, Mikhail G.; Leichtnam, Eric (2013), "Commuting and non-commuting infinitesimals", American Mathematical Monthly, 120 (7): 631-641, arXiv:1304.0583, doi:10.4169/amer.math.monthly.120.07.631.

7 Topology

7.1 Jesper Grodal

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Relevant interests:

Topology, Algebra, Geometry.

Suggested projects:

- GROUP COHOMOLOGY [Alg2]
To a group G we can associate a collection of abelian groups $H^n(G)$, $n \in \mathbf{N}$, containing structural information about the group we started with. The aim of the project would be to define these groups, examine some of their properties, and/or examine applications to algebra, topology, or number theory. See e.g.: K.S. Brown: Cohomology of groups
- GROUP ACTIONS [Top, Alg2]
How can groups act on different combinatorial or geometric objects? Eg. which groups can act freely on a tree? See e.g.: J.-P. Serre: Trees.
- THE BURNSIDE RING [Alg2]
Given a group G we can consider the set of isomorphism classes of finite G -sets. These can be "added" and "multiplied" via disjoint union and cartesian products. By formally introducing additive inverses we get a ring called the Burnside ring. What's the structure of this ring and what does it have to do with the group we started with? See:
http://en.wikipedia.org/wiki/Burnside_ring
- THE CLASSIFICATION OF FINITE SIMPLE GROUPS [Alg2]
One of the most celebrated theorems in 20th century mathematics gives a complete catalogue of finite simple groups. They either belong to one of three infinite families (cyclic, alternating, or classical) or are one of 26 sporadic cases. The aim of the project is to explore this theorem and perhaps one or more of the sporadic simple groups. See:
http://en.wikipedia.org/wiki/Classification_of_finite_simple_groups
- THE PLATONIC SOLIDS AND THEIR SYMMETRIES [Top, Alg2]
A Platonic solid is a convex polyhedron whose faces are congruent regular polygons, with the same number of faces meeting each vertex. The ancient greeks already knew that there were only 5 platonic solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The aim of the project is to understand the mathematics behind this. See: http://en.wikipedia.org/wiki/Platonic_solid

- TOPOLOGICAL SPACES FROM CATEGORIES [Top, Alg2]
Various algebraic or combinatorial structures can be encoded via geometric objects. These "classifying spaces" can then be studied via geometric methods. The goal of the project would be to study one of the many instances of these this, and the project can be tilted in either topological, categorical, or combinatorial directions. See e.g.: A. Bjørner, Topological methods. Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.
- SIMPLICIAL COMPLEXES IN ALGEBRA AND TOPOLOGY [Alg1, Top]
The goal of this project is to understand how simplicial complexes can be used to set up a mirror between notions in topology and algebra. For instance, the algebraic mirror image of a topological sphere is a Gorenstein ring.

Previous projects:

- STEENROD OPERATIONS—CONSTRUCTION AND APPLICATIONS [AlgTopII]
- HOMOTOPY THEORY OF TOPOLOGICAL SPACES AND SIMPLICIAL SETS [AlgTopII]
- AUTOMORPHISMS OF G - WITH APPLICATIONS TO GROUP EXTENSIONS [AlgTopII, CatTop]

7.2 Jesper Michael Møller

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Relevant interests:

All kinds of mathematics.

Suggested projects:

- POINCARÉ SPHERE [Topology, group theory]
What are the properties of the Poincaré sphere?
- TOPOLOGICAL COMBINATORICS [Dis1, Top]
Combinatorial problems, such as determining chromatic numbers of graphs, can be solved using topological methods.
- PARTIALLY ORDERED SETS [Dis1]
Partially ordered sets are fundamental mathematical structures that lie behind phenomena such as the Principle of Inclusion-Exclusion and the Möbius inversion formula.
- CHAOS [General topology]
What is chaos and where does it occur?
- PROJECT OF THE DAY [Mathematics]
<http://www.math.ku.dk/~moller/undervisning/fagprojekter.html>

7.3 Nathalie Wahl

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Relevant interests:

Graphs, surfaces, manifolds, knots, algebraic structures.

Suggested projects:

- **KNOTS [Alg1,Top]**
Mathematically, knots are embeddings of circles in 3-dimensional space. They are rather complicated objects that can be studied combinatorially or via 3-manifolds. The project consists of learning some basics in knot theory. See for example <http://www.earlham.edu/~peters/knotlink.htm>.
- **BRAID GROUPS, CONFIGURATION SPACES AND LINKS [Alg1,Top]**
The braid group on n strands can be defined in terms of braids (or strings), or as the fundamental group of the space of configurations of n points in the plane. It is related to knots and links, and also to surfaces. The project consists of exploring braid groups or related groups like mapping class groups. See for example J. Birman, Braids, links, and mapping class groups.
- **CLASSIFICATION OF SURFACES [Top,Geom1]**
Closed 2-dimensional surfaces can be completely classified by their genus (number of holes). There are several ways of proving this fact and the project is to study one of the proofs. See for example W. Massey, A Basic Course in Algebraic Topology, or A. Gramain, Topology of Surfaces.
- **3-MANIFOLDS [Top,Geom1]**
3-dimensional manifolds are a lot harder to study than 2-dimensional ones. The geometrization conjecture (proved recently by Perelman) gives a description of the basic building blocks of 3-manifolds. Other approaches to 3-manifolds include knots, or “heegaard splittings”, named after the Danish mathematician Poul Heegaard. The project consists of exploring the world of 3-manifolds. See for example <http://en.wikipedia.org/wiki/3-manifolds>.
- **NON-EUCLIDEAN GEOMETRIES [Geom1]**
Euclidean geometry is the geometry we are used to, where parallel lines exist and never meet, where the sum of the angles in a triangle is always 180° . But there are geometries where these facts are no longer true. Important examples are the hyperbolic and the spherical geometries. The project consists of exploring non-euclidian geometries. See for example http://en.wikipedia.org/wiki/Non-euclidean_geometries
- **FROBENIUS ALGEBRAS, HOPF ALGEBRAS [LinAlg,Alg1]**
A Frobenius algebra is an algebra with extra structure that can be described algebraically or using surfaces. A Hopf algebra is a similar structure. Both types

of algebraic structures occur many places in mathematics. The project consists of looking at examples and properties of these algebraic structures. See for example J. Kock, Frobenius algebras and 2D topological quantum field theories.

- KHOVANOV HOMOLOGY [AlgTop – or familiarity with category theory]
The complexity of knots is immense. Explore <http://katlas.org/>. Over the last 100 years various tools have been developed to distinguish and classify knots. A lot of work is still needed to have a good understanding of the world of knots.

This project would aim at understanding one of the stronger tools available to this date; Khovanov Homology.

- OPERADS AND ALGEBRAS [Alg2]
Operads is an effective tool to cope with exotic algebraic structures. How do you for instance work with algebraic structures that are not (strictly) associative? Depending on interest, the project can have a more algebraic or more topological flavor.

- MORSE THEORY [Geom2 – for instance simultaneously]
The second derivative test, known from MatIntro, tells you about local behaviour of a 2-variable function. Expanding this test to manifolds in general yields Morse Theory, which plays a key role in modern geometry.

This project would start out by introducing Morse Theory. Various structure and classification results about manifolds could be shown as applications of the theory.

8 History and philosophy of mathematics

8.1 Jesper Lützen

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Relevant interests:

History of Mathematics

Suggested projects:

- THE HISTORY OF NON-EUCLIDEAN GEOMETRY [Hist1, preferably VtMat]
How did non-Euclidean geometry arise and how was its consistency "proved".
How did the new geometry affect the epistemology of mathematics?
- THE DEVELOPMENT OF THE FUNCTION CONCEPT [Hist1]
How did the concept of function become the central one in mathematical analysis and how did the meaning of the term change over time.
- ARCHIMEDES AND HIS MATHEMATICS [Hist1]
Give a critical account of the exciting life of this first rate mathematician and analyze his "indivisible" method and his use of the exhaustion method.
- WHAT IS A MATHEMATICAL PROOF, AND WHAT IS ITS PURPOSE [Hist1, VtMat]
Give philosophical and historical accounts of the role(s) played by proofs in the development of mathematics

Previous projects:

- A BRIEF HISTORY OF COMPLEX NUMBERS [Hist1, preferably KomAn]
- MATHEMATICAL INDUCTION. A HISTORY [Hist1]
- ASPECTS OF EULER'S NUMBER THEORY [Hist1, ElmTal]
- MATHEMATICS IN PLATO'S DIALOGUES [Hist1, VtMat]
- AXIOMATIZATION OF GEOMETRY FROM EUCLID TO HILBERT [Hist1, preferably VtMat]
- LAKATOS' PHILOSOPHY APPLIED TO THE FOUR COLOR THEOREM [Dis, Hist1]
- HISTORY OF MATHEMATICS IN MATHEMATICS TEACHING: HOW AND WHY [Hist1, DidG preferably DidMat]

8.2 Henrik Kragh Sørensen og Mikkel Willum Johansen

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Relevant interests:

Matematikkens videnskabsteori, matematikkens historie.

Suggested projects:

- DIT EGET PROJEKT? []
I videnskabsteori vil vi typisk i et samarbejde mellem den studerende og underviseren søge at designe individuelle projekter, der passer til den enkelte studerendes faglige profil og interesser. Som nogle eksempler på mulige projektemner kan vi dog nævne de nedenstående:
- COMPUTERASSISTEREDE BEVISER [VtMat]
Computere spiller en stadig stigende rolle i matematisk forskningspraksis. Igennem at studere eksempler på computer-assisterede beviser (fra Firefarvesætningen) og fremad kan man udfordre nogle af vores filosofiske antagelser om matematik. Man har fx diskuteret, hvorvidt matematisk viden, som baserer sig på computerbeviser, stadig er a priori viden.
- ALGEBRA I 1800-TALLET [Hist1, VtMat]
I løbet af 1800-tallet blev algebraen udviklet fra en teori om ligningsløsning til en generel teori om algebraiske strukturer. Igennem et nedslag i fx Gauss' studier af hvornår cirkelperiferien kan inddeles i n lige store stykker med passer og lineal (1801), Abels bevis for femtegradsligningens algebraiske uløselighed (1826), eller gruppebegrebets aksiomatisering i anden halvdel af 1800-tallet kan vi historisk og videnskabsteoretisk diskutere matematikkens udvikling.
- KILDECENTRERET MATEMATIKHISTORIE I GYMNASIET [VtMat, Hist1, DidMat]
Hvordan kan man bruge autentiske matematiske kilder i gymnasiets matematikundervisning og til tværfaglig undervisning? Hvordan skal kilden forberedes? Hvilke undervisningsmål kan sådan en tilgang være med til at opfylde?
- ØKONOMISKE MODELLER []
Diskuter muligheden for at opstille værdifri modeller i økonomien. Diskuter resultatet fx i relation til værdi-fakta-skellet i samfundsdebatten.

Litteraturforslag: Hausman & McPherson: *Economic analysis, moral philosophy, & public policy*; Klemens Kappel: *Videnskabens særlige rolle i det liberale demokrati*
- KOGNITIV SEMANTIK []
Beskriv G. Lakoff & R. Núñez' forsøg på at give matematikken et grundlag i menneskets biologi og kognition. Diskuter rimeligheden af dette grundlag fx

ved at inddrage og undersøge cases eller eksempler taget fra dine andre matematikkurser.

Litteraturforslag: Lakoff & Núñez: *Where mathematics comes from*.

- SYMBOLER OG DIAGRAMMER I MATEMATIKKEN []
Den moderne matematiske praksis er intimt sammenkædet med brugen af højt specialiserede symboler og diagrammer. Beskriv en eller flere filosofiske teorier om brugen af den type repræsentationer. Det kan fx være teorier som C.S. Peirces teori om diagrammatisk tænkning eller den mere moderne teori om distribueret kognition. Analyser med udgangspunkt i den valgte teori et eller flere eksempler på brug af diagrammer og symboler taget fra dine øvrige kurser. Vurder og diskuter på den baggrund validiteten af den valgte teori.

Litteraturforslag: Hutchins: *Material anchors for conceptual blends*; Kirsh: *Thinking with external representations*; De Cruz: *Mathematical symbols as epistemic actions – an extended mind perspective*; Peirce: *Polegomena for an apology to pragmatism*.

- ALAIN CONNES' PLATONISME []
Den franske matematiker Alain Connes er et af de bedste eksempler på en moderne platonisk i matematikken. Beskriv Alain Connes' bud på en moderne platonisme og diskuter med inddragelse af relevant filosofisk teori i hvilken grad projektet lykkes.

Litteraturforslag: Changeux & Connes: *Conversations on mind, matter, and mathematics*.

- ØKONOMISKE MODELLERS PERFORMATIVITET []
Diskuter, fx med udgangspunkt i Black-Scholes-modellen i hvilken grad og gennem hvilke mekanismer matematiske modeller kan påvirke og ændre samfundet.

Litteraturforslag: MacKenzie: *An Equation and its Worlds: Bricolage, Exemplars, Disunity and Performativity in Financial Economics*; Callon: *The laws of the markets*.

- DOMÆNEUDVIDELSE I MATEMATIKKEN []
Gennem en analyse af Hamiltons udvidelse af komplekse tal til kvaternionerne opstiller Andrew Pickering en mekanisme, der skal forklare hvordan domæneudvidelser generelt foregår i matematikken. Beskriv Pickerings mekanisme og diskuter hvor rimelig den er på baggrund af analyser af lignende cases.

Litteraturforslag: Pickering: *The mangle of practice*.

9 Teaching and Didactics of Mathematics

9.1 Carl Winsløw

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Relevant interests:

Didactics of Mathematics

Suggested projects:

- MATHEMATICAL CONTENT ANALYSIS AND DESIGN [One or more B.Sc.-courses in mathematics ; DidG]

You select a topic (result, concept, object,...) from secondary mathematics, about which you have acquired significant new knowledge during your bachelor studies in mathematics. Your paper should then include two parts:

1. A concise exposition on the topic, based on central results and methods from one or more courses from the B.Sc. programme in mathematics (excl. MatIntro).
2. Design and theoretical analysis of a didactic situation for this topic (at secondary level).

Here are some example of suitable topics:

- angles
- exponential functions
- linear regression
- \mathbb{R}

It is of course much better if you have another and more exciting idea yourself.

9.2 Other projects

Other projects in this area can be found with

- Niels Grønbæk (6.2)
- Jesper Lützen (8.1)
- Henrik Kragh Sørensen og Mikkel Willum Johansen (8.2)

10 Set Theory

10.1 David Schrittesser

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Relevant interests:

Mathematical logic, descriptive set theory, large cardinals, forcing, inner models.

Suggested projects:

- FORCING []
Forcing is a versatile tool for proving independence results, i.e. results that say a certain theorem is not provable in ZFC (the standard axiom of set theory). The project consists of learning the basics of this technique, either using partial orders or Boolean valued models. A goal could be to prove the independence of the continuum hypothesis or various combinatorial principles, in a reasonably self-contained manner (perhaps but not necessarily black-boxing some basic theorems from standard texts).
- LARGE CARDINALS []
Large cardinals are ubiquitous in modern set theory. They may be called ‘Axioms of strong infinity’: they can be added to the list of the usual axioms of set theory (ZFC or the Zermelo-Fraenkel axiom system with the axiom of choice) and can be interpreted as stating (in one way or another) that there exist very, very large sets. Most large cardinal axioms can also be construed as saying there exists a non-trivial map from the universe to itself which preserves the truth of statements (a set theoretical morphism, if you wish). There are many possible goals for this project: for instance, to describe the ultrapower construction to construct an elementary embedding; or to compare some of the well-known smaller large cardinals in strength.
- INFINITE GAMES AND SETS OF REAL NUMBERS []
Infinite games can be used to show that a given set of reals has desirable properties, like being Lebesgue measurable or having the Baire property, provided one of the players has a winning strategy (i.e. the game is determined). Unfortunately, in general only Borel sets are determined; this gives us that analytic sets (and more) are regular. Modern set theory considers strong axioms of determinacy, with which you can show that more sets are regular. The goal of this project could be to relate determinacy of games to regularity properties such as measurability, the Baire property and the perfect set property. A more ambitious goal would be to prove determinacy of Borel games.

Previous projects:

- FORCING AND SHEAVES []

- GÖDEL'S CONSTRUCTIBLE UNIVERSE AND LARGE CARDINALS []

10.2 Asger Tornquist

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Relevant interests:

Mathematical logic, descriptive set theory, recursion theory, descriptive combinatorics, ergodic theory.

Suggested projects:

- RECURSION THEORY []
 Recursion theory, also called computability theory, is concerned with the study of computable functions on the natural numbers. There are many ways of defining computable functions, the most famous being via so-called Turing machines.
 A project in this area could focus on building up the elementary tools of recursion theory, and then move on to more advanced topics such as relative computability, recursively enumerable degrees, or classical decision problems.
- DESCRIPTIVE SET THEORY AND ERGODIC THEORY []
 Descriptive set theory is the study of the definable sets and functions that could (and do) arise in analysis. The most tangible class of such sets and functions are the Borel sets and functions, known from measure theory.
 On the other hand, ergodic theory is the study of measure-preserving group actions on measure spaces, and has its origin in statistical mechanics in physics.
 A lot of work in descriptive set theory these days focusses on groups, their actions, and their “orbit equivalence relations”, and this has produced a growing overlap of interested with ergodic theory. A project in this area could focus on ergodic theoretic results such as Dye’s theorem and orbit equivalence rigidity, or it could focus on descriptive set theoretic dichotomies, and “orbit cardinals” associated to groups actions.
- RAMSEY THEORY, FINITE AND INFINITE []
 Ramsey theory is a theory of to high-dimensional pigeonhole principles. One way of stating the classical pigeonhole principle is to say: If $n > m$ objects are coloured in m colours, there must be two objects that get the same colour. Ramsey theorists say that there is a “monochromatic” set of size at least 2.
 Somewhat surprisingly, there are higher dimensional versions of this: For instance, if 6 random people walk into a room, then there are 3 people who either all knew each other in advance, or there are 3 people who didn’t know each other in advance. That’s a special case of the classical Ramsey theorem.
 Ramsey theory, a major area of modern combinatorics, is about the various higher-dimensional pigeonhole principles, as well as their infinitary analogues. A project in this area could start with the classical finite Ramsey theorem, and other finite analogues, and then perhaps move into the more challenging setting

of infinitary Ramsey theory, which uses ideas from descriptive set theory and topology.

Previous projects:

- RECURSION THEORY []
- BANACH-TARSKI'S PARADOX WITH BAIRE MEASURABLE PIECES []
- HJORTH'S TURBULENCE THEORY []
- ERGODIC THEORY AND DYE'S THEOREM []
- GÖDEL'S CONSTRUCTIBLE UNIVERSE L []
- INFINITARY COMBINATORICS IN L []
- MARTIN'S AXIOM AND APPLICATIONS []

11 Applied mathematics

11.1 Carsten Wiuf

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Relevant interests:

Markov chains applied to biochemical processes, Simulation, ODEs.

Suggested projects:

- MATHEMATICAL ANALYSIS OF BIOCHEMICAL SYSTEMS AT AND NEAR EQUILIBRIUM. [Diff, Stok]

The project consists of a theoretical part and an applied part. The theoretical part consists in learning how to describe a system as a Markov chain (stochastic) and a system of ODEs (deterministic). The applied part of the project analyzes a biochemical system as a Markov chain and an ODE system and contrasts the results with each other. The biochemical system will be chosen from the literature together with the student. The applied part involves implementation of small scripts in R or Maple.

12 Probability

12.1 Ernst Hansen

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Relevant interests:

Probability, analysis, statistics

Suggested projects:

- GEOMETRIC MEASURE THEORY [MI]
There are a number of interesting results on the boundary between measure theory and geometry: the coarea formula on decomposition of integrals (polar integration decomposes a planar integral as integrals over circles - the coarea formula generalises this to a variety of other decompositions), Sard's theorem on the measure of critical images, Stoke's theorem (allows us to replace the integral over a set A with an integral over ∂A , thus generalising the fundamental theorem of calculus). A project could focus on smooth versions of some of these theorems, based on the notion of a Lebesgue measure on smooth subsets of \mathbb{R}^k . Or it could focus on more general versions of the theorems, based on the notion of a Hausdorff measure on an arbitrary subset of \mathbb{R}^k .
- REFINEMENTS OF CLT [Stok2]
The central limit theorem is a cornerstone in probability theory. It says that properly normalized sums of independent random variables converge weakly to a Gaussian limit. A number of important results elaborate on this convergence: Berry-Esseen's theorem gives explicit bounds for the convergence, Edgeworth expansions and saddlepoint-approximations improve on the convergence (sometimes quite dramatically) by modifying the Gaussian limit with small adjustments. Most of these results are based on a very careful analysis of the characteristic function.
- POISSON APPROXIMATIONS [MI]
Sums of Bernoulli variables are often remarkably well approximated by Poisson distributions. If the Bernoulli variables are iid, the sum has a binomial distribution, and it is easy to see that binomial and Poisson distributions are close. But Poisson approximations can be shown to be valid in a vast variety of non-iid situations as well.
- ISOPERIMETRIC INEQUALITIES [MI]
The classical isoperimetric inequality states that among the closed planar curves with a specified length, a circle will maximize the area of the enclosed domain. This was 'known' since the ancient Greeks, but a rigorous proof was not obtained until the late 19th century. Today, quite accessible proofs can be given using measure theory. A number of generalizations are possible, for instance to spheres (instead of the plane). Or the project could investigate Gaussian isoperimetric

inequalities, where 'length' and 'area' with respect to Lebesgue measure are replaced by similar notions for Gaussian measures. The Gaussian isoperimetric inequality are the gateway to probability theory in Banach spaces.

- ROBUST STATISTICS [Stat2]

It can be shown in considerable generality that the asymptotically best estimators are obtained by maximizing the likelihood function. But this requires that the model is specified correctly. Maximizing the wrong likelihood can lead to quite disastrous results, even if the model is only slightly wrong. For instance, the MLE is in general very sensitive to the presence of 'outliers', which can be thought of as observations that do not conform to the model. Robust statistics are methods that try to safe-guard against minor deviations from the model.

13 Mathematics of Quantum Theory

13.1 Matthias Christandl

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Relevant interests:

Quantum Information Theory and Quantum Computation

Suggested projects:

- BELL INEQUALITIES [QIT1]
Quantum mechanics violates Bell's inequalities, which are inequalities that classical local structures satisfy. When the violation is very high, a stronger notion, known as *rigidity* can appear, where it is possible to infer the structure of measurements and states that have been used to infer the violation. The project will investigate rigidity, taking its starting point in a famous theorem by Tsirelson.
- QUANTUM ENTROPY [QIT2]
Quantum entropy or von Neumann entropy is a concave function of the quantum state. Interestingly, there are more concavity-like properties that quantum entropy satisfies. It is the aim of the project to study these properties.
- QUANTUM TOMOGRAPHY [QIT3]
Quantum tomography is the art of inferring a quantum state from measured data. To a large extent, quantum tomography is a problem from classical statistics and it is the goal to collect classical methods to treat this problem.

13.2 Jan Philip Solovej

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Relevant interests:

Mathematical Physics, Quantum Mechanics, Spectral Theory, Partial Differential Equations

Suggested projects:

- WHY YOU CANNOT HEAR THE SHAPE OF A DRUM [An1,An2]
For any compact open set in \mathbb{R}^n we can define the discrete set of eigenvalues of the Dirichlet Laplacian. In the 2-dimensional case these are the frequencies you would hear if the domain is played as a drum. Translating, rotating or reflecting a domain does not change the eigenvalues. But is this the only way domains can have the same frequencies? I.e., can you hear the shape of a drum? The answer is no and the project constructs pairs of domains in \mathbb{R}^2 that have the same Dirichlet Laplace eigenvalues, but are not isometric. As part of the project

the existence of the Dirichlet Laplace eigenvalues will be constructed. A certain regularity property of eigenfunctions will have to be used, but not proved.

Literature: Notes and exercises and the article Buser, Conway, Doyle, and Semmler, Some planar isospectral domains, IMRN, 1994 (9)

- POISSON SUMMATION FORMULA AND GAUSS CIRCLE PROBLEM [An1]
The aim is to prove the Poisson summation formula using the theory of Fourier series. The Poisson summation formula can be used to estimate how close Riemann sums are to Riemann integrals. This will be generalized to functions of several variables and applied to count the integer points in a large ball (Gauss circle problem). The problem is of interest in number theory and in quantum mechanics, where it may be interpreted as the number of states in a Fermi gas below the Fermi level.
- ESTIMATING EIGENVALUES OF SCHRÖDINGER OPERATORS [An1,An2]
The goal of this project is to estimate the number and the sum of negative eigenvalues for Schrödinger operators. In quantum mechanics the negative eigenvalues of Schrödinger operators represent bound states of quantum systems and their sum represents the total energy of the bound states. This project requires first to introduce the Fourier transform

13.3 Niels Benedikter

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Relevant interests:

Mathematical Physics, Many-Body Quantum Mechanics, Non-Relativistic Quantum Electrodynamics, Spectral Theory

Suggested projects:

- EFFECTIVE EVOLUTION EQUATIONS FROM MANY-BODY QUANTUM MECHANICS [MathPhys, DiffFun, Quantum Mechanics or Functional Analysis recommended but not strictly necessary]
The time-dependent Schrödinger equation describes the time-evolution of quantum mechanical many-body systems. However, since it is an extremely complicated equation, it is a central problem in mathematical physics to rigorously derive simpler effective evolution equations that describe the main physical properties in specified physical situations. These include the Hartree and Gross-Pitaevskii equation which describe Bose-Einstein condensates or the time-dependent Hartree-Fock equation which describes e. g. processes in the atomic nucleus.

In this project the goal is to understand the proof of the validity of an effective equation and work on an improvement of the derivation, e. g. to include some effects of relativity.

[BPS] N. Benedikter, M. Porta, B. Schlein: *Effective Evolution Equations from Quantum Dynamics*, SpringerBriefs in Mathematical Physics, 2015
<https://arxiv.org/abs/1502.02498>

- HARTREE-FOCK THEORY [MathPhys, DiffFun, Quantum Mechanics or Functional Analysis recommended but not strictly necessary]
Effective equations are not only important for the time evolution of quantum mechanical many-body systems, but also to understand the energy levels. In this project, we will work through some classical papers on the derivation of the time-independent Hartree-Fock equations and write a detailed proof. If time permits, relations to e.g. time-dependent effective equations may be studied or some open problems in the field can be discussed.

[GS] G. M. Graf, J.P. Solovej: A correlation estimate with applications to quantum systems with Coulomb interactions, *Reviews in Mathematical Physics*, 1994
https://www.ma.utexas.edu/mp_arc/c/93/93-60.ps.gz

- SPECTRAL THEORY FOR A MODEL OF BIOLOGICAL MEMBRANES [MathPhys, DiffFun or Functional Analysis or similar highly recommended]
In this project we consider some models describing biological membranes coupled to the cell's cytoskeleton. The spectral theory of the corresponding operators can be studied in analogy to well-known operators from mathematical quantum mechanics. The goal of the project is to learn the well-known techniques and extend them to understand boundary conditions and self-adjoint realizations, and possibly obtain some more quantitative information about the spectrum.

[RS] M. Reed, B. Simon: *Methods of Modern Mathematical Physics, Vol 2: Fourier Analysis, Self-Adjointness*, Academic Press, 1975
<http://bookzz.org/>

13.4 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- QUANTUM FIELD THEORY IN 1-D: THE CASIMIR EFFECT [An1, An2, KomAn]
As you may have heard, in a certain sense $1 + 2 + 3 + 4 + \dots = -1/12$. Properly framed, this even has a direct physical relevance and can be measured in the laboratory as the attractive force (in appropriate units) between two conductors in a vacuum, which is known as the Casimir effect. It also makes sense mathematically because it turns out that there is a canonical way of regularizing the sum to do away with the infinities. One popular approach is via spectral

zeta functions for Schrödinger operators, which includes Riemann's zeta function $\zeta(s)$. You can in this project look at simple 1-dimensional “toy” quantum field theories, study different regularization methods and among other things show that the value “ $-1/12$ ” is well-defined.

14 Other areas

14.1 Discrete mathematics

Projects in this area can be found with

- Bergfinnur Durhuus (4.1)
- Søren Eilers (6.1)
- Mikael Rørdam (6.5)

14.2 Aspects of computer science

Projects in this area can be found with

- Søren Eilers (6.1)