Bachelor projects for mathematics and mathematics-economics

Department of Mathematical Sciences
University of Copenhagen

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Contents

Introduction

This is a catalogue of projects suggested by the researchers at the Department of Mathematical Sciences for students in the B.S. programs in mathematics and mathematics-economics. It is important to note that such a catalogue will never exhaust all possibilities – indeed, if you are not finding what you are looking for you are strongly encouraged to ask the member of our staff you think is best qualified to help you on your way for suggestions of how to complement what this catalogue contains. Also, the mathematics-economics students are encouraged to study the searchable list of potenial advisors at the Economy Department on

`http://web.econ.ku.dk/polit/studerende/Speciale/vejlederoversigt/`.

If you do not know what person to approach at the Department of Mathematical Sciences, you are welcome to try to ask

- the director of studies (Ernst Hansen, erhansen@math.ku.dk) or
- the assocate chair for education (Tinne Hoff Kjeldsen, thk@math.ku.dk).

When you have found an advisor and agreed on a project, you must produce a contract (your advisor will know how this is done), which must then be approved by the director of studies at the latest during the first week of a block. The project must be handed in during the 7th week of the following block, and an oral defense will take place during the ninth week.

We wish you a succesful and engaging project period!

Best regards,

Tinne Hoff Kjeldsen	Ernst Hansen
Associate chair	Director of studies

1 Finance

1.1 Rolf Poulsen and David Skovmand

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Relevant interests:

Finance.

Suggested projects:

- Option pricing [Fin1]
  Pricing and hedging of exotic options (barrier, American (Longstaff & Schwartz (2000)), cliquet). A detailed investigation of convergence in of the binomial model (papers by Mark Joshi). Model calibration as an inverse problem (Derman & Kani (1995)). Incomplete models; equilibrium pricing (Carr & Madan (2000), Garleanu, Pedersen & Allen Poteshman (2009)).

- Stochastic interest rates [Fin1]
  Yield curve estimation. Estimation of dynamic short rate models. Calibration and the forward algorithm. Derivative pricing with applications to embedded options in mortgage products, the leveling algorithm. Risk management for mortgagors and pension funds.

- Optimal portfolio choice [Fin1]
  The effects of parameter uncertainty on optimal portfolio choice (Kan & Zhou (2007) for instance). *Betting Against Beta* (Frazzini & Pedersen (2014)) and other CAPM-related stuff. Optimal multi-period investment with return predictability and transaction costs (Garleanu & Pedersen (2013))´.

- Stochastic volatility [Fin1 and preferably, but not necessarily, FinKont]
  What is volatility? (There are at least a handful of definitions that all more or less are the same in the Black-Scholes model – but not in more general models.) How does volatility behave empirically? How does (stochastic) volatility affect (plain vanilla) option pricing? How can variance swaps (and other volatility derivatives) be priced and hedged?

- Other projects [?]
  By following the link `https://tinyurl.com/y8al3usr` you can get a wealth of other project suggestions (in finance and OR).

2 Operations research

2.1 Trine Krogh Boomsma

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Relevant interests:

Operations Research

Suggested projects:

- An integer programming approach to university timetabling [OR1, possibly Applied Operations Research/OR2]
  The problem is to construct a timetable for the scheduling of university courses to time slots, rooms and teachers. All courses must be allocated the required number of hours per week and assigned a room of appropriate size and a teacher who covers the topic. A timetable must take into account that some classes require a number of subsequent time slots, while there must be no overlapping of mandatory courses and courses on related topics. Moreover, courses must have the same teacher in all time slots and, as far as possible, the same room. The timetabling problem can be formulated by integer programming (IP), with binary variables indicating whether a specific course is allocated to a specific time slot, room and teacher. Real data is available from the Faculty of Law, University of Copenhagen. Project work involves the formulation of objective and constraints, processing of data, software implementation of the problem, e.g. in GAMS, and discussion of its solution.
  Relevant literature: S. Kristiansen and T. J. R. Stidsen, A comprehensive study of educational timetabling - a survey. European Journal of Operational Research 73(1), 2013. S. Daskalaki, T. Birbas, and E. Housos, An integer programming formulation for a case study in university timetabling, European Journal of Operational Research, 153(1), 2004.

- Electricity generation scheduling by dynamic programming [OR1, possibly Applied Operations Research/OR2]
  With the increasing share of renewable energy sources in most electricity systems, the scheduling and valuation of flexible power generation becomes more and more important. More specifically, since renewable energy production is often rather inflexible but highly fluctuating, the balance between supply and demand depends on conventional generation being able to adjust accordingly. The present project aims to value flexible power generation, while taking into account its ability to adjust production over time. The challenge is to formulate the valuation problem by (stochastic) dynamic programming in a way that accounts for operational constraints. Project work involves the formulation of the unit commitment problem, the application of dynamic programming, software implementation and discussion of results.
  Relevant literature: Stochastic power generation unit commitment in electricity

markets: A novel formulation and a comparison of solution methods, Operations Research 57(1), 2009. Short-term generation asset valuation: A real options approach, Operations Research 50(2), 2002.

2.2 Giovanni Pantuso

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Relevant interests:

Operations Research

Suggested projects:

- OR for Urban Transportation [OR1, possibly Applied Operations Research]
  Shared and sustainable/electrified mobility is slowly reshaping urban mobility. Particularly, car-sharing and bike-sharing services are gaining increasing popularity in most major cities around the globe. Such services generate new and complex planning problems such as the positioning, relocation, pricing and charging of shared bicycles or cars. Addressing such planning problems requires competencies in OR. This project consists of studying and modeling a specific planning problem faced in car/bike-sharing services. The work required in this project includes: reading the relevant literature, obtaining, analyzing and preparing data, formulating and solving mathematical programming problems, discussing the results. Relevant literature:

  - N. Juul, G. Pantuso, J.E.B. Iversen, T.K. Boomsma (2015) *Strategies for Charging Electric Vehicles in the Electricity Market*, International Journal of Sustainable Energy Planning and Management 7, 67-74
  - R.G. Hansen, G. Pantuso (2018) *Pricing Car-Sharing Services in Multi-Modal Transportation Systems: An Analysis of the Cases of Copenhagen and Milan*. In: Cerulli R., Raiconi A., Voß S. (eds) Computational Logistics. ICCL 2018. Lecture Notes in Computer Science, vol 11184. Springer, Cham

- OR in Sports [OR1, possibly Applied Operations Research]
  Sports analytics is becoming widely popular. Many professional teams and individual player use mathematical tools to make smarter decisions and improve performance. This project consists of using OR techniques to improve current decisions in team-sports, such as deciding which players to use, how to position them on the field, and how to evaluate their performance. The work required in this project includes: reading the relevant literature, obtaining, analyzing and preparing data, formulating and solving mathematical programming problems, discussing the results. Relevant literature:

  - Michael J. Fry, Jeffrey W. Ohlmann, (2012) *Introduction to the Special Issue on Analytics in Sports, Part I: General Sports Applications*. Interfaces 42(2):105-108. http://dx.doi.org/10.1287/inte.1120.0633

– Michael J. Fry, Jeffrey W. Ohlmann, (2012) *Introduction to the Special Issue on Analytics in Sports, Part II: Sports Scheduling Applications.* Interfaces 42(3):229-231. http://dx.doi.org/10.1287/inte.1120.0632

– G. Pantuso, (2017) *The Football Team Composition Problem: a Stochastic Programming approach.* Journal of Quantitative Analysis in Sports 13 (3), 113-129

3 Algebra and number theory

3.1 Henrik Holm

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Relevant interests:

Rings, modules, homological algebra, category theory.
The prerequisites for the following projects are the courses [Alg 1] and [Alg 2]. Details, and possibly additional suggestions for projects, may be found at my homepage *http://www.math.ku.dk/~holm/*

Suggested projects:

- COMPLETION OF RINGS [Alg1, Alg2]
  Given an ideal I in a commutative ring R one can construct the I-adic completion $\widehat{R}_I$. For example, $\widehat{k[x]}_{(x)}$ is the formal power series ring and $\widehat{\mathbb{Z}}_{(p)}$ is the ring of p-adic integers. The aim of this project is to define adic completions and to investigate their basic properties.
  Literature: H. Matsumura, "Commutative ring theory".

- GROUP (CO)HOMOLOGY [Alg1, Alg2]
  To a group G one can associate a sequence of (abelian) homology groups $H_n(G)$ and cohomology groups $H^n(G)$ that contain information about G. For example, $H_1(G) = G_{\mathrm{ab}}$ is the abelianization of G. The aim of this project is to define group (co)homology and to give group theoretical descriptions of the lower (co)homology groups.
  Literature: P. J. Hilton and U. Stammbach, "A course in homological algebra".

- GRÖBNER BASES [Alg1, Alg2]
  Given an ideal I in the polynomial ring $k[x_1, \ldots, x_n]$ and a term ordering $\leqslant$ one can always find a so-called Gröbner basis $g_1, \ldots, g_m$ of I with respect to $\leqslant$. For example, a Gröbner basis for the ideal $I = (y^2 - x^3 + x, y^3 - x^2)$ with respect to the lexicographic ordering (where $x \geqslant y$) consists of $g_1 = y^9 - 2y^6 - y^4 + y^3$ and $g_2 = x - y^7 + y^4 + y^2$. Gröbner bases are powerful tools to solve e.g. polynomial equations and the ideal membership problem. The aim of this project is to define, and to show the existence of, Gröbner bases, and to demonstrate some applications.
  Literature: N. Lauritzen, "Concrete abstract algebra" and D. Cox, J. Little, and D. O'Shea, "Ideals, varieties, and algorithms".

- INJECTIVE MODULES [Alg1, Alg2]
  An object in a category is called injective if it has a certain lifting property. For example, the injective objects in the category of abelian groups are precisely the divisible abelian groups (such as the group of rational numbers $\mathbb{Q}$ and the Prüfer groups $\mathbb{Z}(p^\infty)$ where p is a prime). The aim of this project is to develop the theory of injective modules over an arbitrary ring.

Literature: F. W. Anderson and K. R. Fuller, "Rings and categories of modules" and E. E. Enochs and O. M. G. Jenda, "Relative homological algebra".

- THE LOWER K-GROUPS OF A RING [Alg1, Alg2]
  The algebraic K-theory of a ring R is a certain sequence $K_n(R)$ of abelian groups that contains information about R. For example, if R is a field, then $K_0(R) = \mathbb{Z}$ is the additive group of integers and $K_1(R) = R^\times$ is the multiplicative group of units in R. The aim of this project is to define and investigate the lower K-groups for certain classes of rings.
  Literature: J. Rosenberg, "Algebraic K-theory and its applications".

- ADJOINT FUNCTORS IN CATEGORY THEORY [Alg1, Alg2]
  Adjoint functors are important and abundant in mathematics. For example, the forgetful functor $U\colon \mathbf{Vct} \to \mathbf{Set}$ from the category of (real) vector spaces to the category of sets has a left adjoint $V\colon \mathbf{Set} \to \mathbf{Vct}$, which to each set X associstes the (real) vector space with basis X. The aim of this project is to develop the basic theory of adjoint functors and to prove Freyd's Adjoint Functor Theorem and The Special Adjoint Functor Theorem.
  Literature: S. Mac Lane, "Categories for the working mathematician".

3.2 Christian U. Jensen

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Relevant interests:

Galois theory. Algebraic number theory.

Suggested projects:

- INTRODUCTORY GALOIS THEORY [Alg2]
  This is the study of roots of polynomials and their symmetries: one studies the fields generated by such roots as well as their associated groups of symmetries, the so-called Galois groups. Galois theory is fundamental to number theory and other parts of mathematics, but is also a very rich field that can be studied in its own right.

- INTRODUCTION TO ALGEBRAIC NUMBER THEORY [Alg2]
  Algebraic number theory studies algebraic numbers with the main focus on how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers and the study leads to a lot of new theories and problems. The study is necessary for a lot of number theoretic problems and has applications in many other parts of mathematics.

3.3 Ian Kiming and Fabien Mehdi Pazuki

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Relevant interests:

Algebraic number theory and arithmetic geometry

Suggested projects:

- ALGEBRAIC NUMBER THEORY AND FERMAT'S LAST THEOREM [Alg2]
  The aim of the project is to get an introduction to algebraic number theory and then apply this to the study of Fermat's last theorem for so-called regular primes. Algebraic number theory studies how to generalize the notion of integers and their prime factorizations. This turns out to be much more complicated for general systems of algebraic numbers, but is essential for the study of many number theoretic problems such as Fermat's last theorem. Concretely, the project will aim at understanding the definition of a regular prime p and the proof of Fermat's last theorem in the so-called first case: this is the statement that the equation $x^p + y^p + z^p = 0$ does not have any non-trivial solution in integers x, y, z with $p \nmid xyz$.

- p-ADIC NUMBERS [Alg2]
  The real numbers arise from the rational numbers by a process called "completion". It turns out that the rational numbers (and more generally any algebraic number field) has infinitely many other "completions", – one for each prime number p. Given p, the corresponding field is called the field of p-adic numbers. These fields have a lot of applications in many branches of mathematics, not least in number theory for instance in the theory of Diophantine equations, i.e., the question of finding integral (or rational) solutions to polynomial equations with integer coefficients.

- HASSE-MINKOWSKI'S THEOREM ON RATIONAL QUADRATIC FORMS [Alg2]
  A rational quadratic form is a homogeneous polynomial of degree 2 and with rational coefficients, for example $x^2+xy+3y^2+yz+2z^2$. The Hasse-Minkowski theorem states that such a polynomial has a non-trivial rational zero if and only if it has a non-trivial zero in the field of real numbers as well as in all fields of p-adic numbers. The latter condition can be translated into a finite number of congruence conditions modulo certain prime powers and one thus obtains an effective (algorithmic) criterion. The project includes an initial study of p-adic numbers.

- CONTINUED FRACTIONS AND PELL'S EQUATION [Alg2]
  The project studies the theory of continued fractions and how this theory can be applied to determining so-called units in quadratic number fields. This has applications to the study of Pell (and "non-Pell") equations, i.e., the problem of solving in integers equations of the form $x^2 - Dy^2 = \pm 1$ where D is a given positive, squarefree integer.

- CLASS GROUPS OF QUADRATIC FIELDS AND BINARY QUADRATIC FORMS [Alg2]
  A quadratic number field is a field obtained form $\mathbb{Q}$ by adjoining a number of

the form $\sqrt{D}$ with D an integer that is not a square in $\mathbb{Z}$. The class group attached to such a field measures how far its so-called ring of integers is from being a unique factorization domain. These class groups can be used as a tool in studying integer solutions to equations of the form $ax^2 + by^2 = c$ for given integers a, b, c.

- Introduction to modular forms [Alg2, KomAn]
  The project is an introduction to modular forms, especially on $\mathrm{SL}_2(\mathbb{Z})$. These are initially complex analytic objects, and thus a certain, minimal background in complex analysis is required. Modular forms turn out to have a lot of deep connections to arithmetic, and one can use this project as a platform for a later study of the more general modular forms on congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$. These are very important in modern number theory and are for instance central in Andrew Wiles' proof of Fermat's last theorem.

- Elliptic curves and Mordell's theorem [Alg2]
  The project is an introduction to elliptic curves defined over $\mathbb{Q}$. These are curves given by equations of the form $y^2 = x^3 + ax^2 + bx + c$ with rational a, b, c (such that $x^3 + ax^2 + bx + c$ is without multiple roots). The study of elliptic curves is one of the central themes in modern number theory. It turns out that the set of rational points on an elliptic curve, that is, the set of rational solutions to the equation (+ a "point at infinity") can be given a natural structure as abelian group. This is called the group of rational points and can be finite or infinite depending on the curve. Mordell's famous theorem is that the group of rational points is a finitely generated abelian group.

- Elliptic curves and the theorem of Billing-Mahler [Alg2]
  The project will include an introduction to elliptic curves as above and then proceed towards a proof of the theorem of Billing-Mahler. A big theorem of Barry Mazur (1977) implies that if n is the order of a rational point of finite order on an elliptic curve defined over $\mathbb{Q}$ then either $1 \leq n \leq 10$ or $n = 12$. In particular, $n = 11$ is impossible. This latter statement is the theorem of Billing and Mahler (1940). The proof will also involve a bit of algebraic number theory. The impossibility of $n = 13$ can also be proved by these methods.

- Torsion points on elliptic curves [Alg2]
  The project will include an introduction to elliptic curves as above and then proceed to study rational torsion points, i.e., rational points of finite order, on elliptic curves defined over $\mathbb{Q}$. There are several possibilities here, such as parametrizations of curves with a point of given, low order, (generalizations of) the theorem of Lutz-Nagell, the structure of the group of torsion points on elliptic curves defined over a p-adic field (Lutz' theorem).

- Primality testing [Alg2]
  How can one decide efficiently whether a large number is a prime number? The project will study mathematically sophisticated methods of deciding this: the

Miller-Rabin probabilistic primality test and the more recent Agrawal-Kayak-Saxena deterministic primality test. The project will include an initial study of algorithmic complexity theory as will as a bit of the theory of finite fields.

- OPEN PROJECT [?]
  If you have some ideas of your own for a project within the general area of number theory, you can always come and discuss the possibilities with us. You can also come and discuss if you do not have specific ideas of your own, would like a project in number theory, but not any of the above.

Previous projects:

- NON-EXHAUSTIVE LIST OF PREVIOUS PROJECTS: []
- THE j-INVARIANT OF ELLIPTIC CURVES [Alg2]
- REPRESENTATION OF INTEGERS BY BINARY QUADRATIC FORMS [Alg2, Algnum]
- PRIMALITY TESTING IN POLYNOMIAL TIME [Alg2]
- DIRICHLET'S UNIT THEOREM [Alg2, Algnum]
- BERNOULLI NUMBERS, REGULAR PRIMES, AND FERMAT'S LAST THEOREM [Alg2]
- ALGEBRAIC FUNCTION FIELDS [Alg2]
- GALOIS GROUPS OF POLYNOMIALS OF DEGREE FIVE [Alg3]
- p-ADIC NUMBERS: CONSTRUCTION AND APPLICATIONS [Alg2]
- THE STRUCTURE OF THE ALGEBRA OF MODULAR FORMS OF LEVEL 1 MODULO p [Alg2, KomAn]
- THE THEOREM OF HASSE-MINKOWSKI [Alg2]
- THE THEOREM OF BILLING-MAHLER [Alg2]
- ELLIPTIC CURVES AND SELMER GROUPS [Alg3]
- TORSION POINTS ON ELLIPTIC CURVES [Alg2]

3.4 Other projects

Other projects in this area can be found with

- Jesper Grodal (7.1)
- Morten S. Risager (4.4)

4 Analysis

4.1 Bergfinnur Durhuus

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Relevant interests:

Analysis: Operator theory, differential equations. Mathematical physics: Quantum mechanics, statistical mechanics. Discrete mathematics: Graph theory, analytic combinatorics, complexity theory,

Suggested projects:

- GRAPH COLOURING PROBLEMS [Dis1, An1]
  Problems originating from various areas of mathematics can frequently be formulated as colouring problems for certain types of graphs. The four-colour problem is probably the best known of coulouring problems but there is a variety of other interesting colouring problems to attack

- COMBINATORICS OF GRAPHS [Dis1, An1, ComAn]
  Counting of graphs specified by certain properties (e.g. trees) is one of the classical combinatorial problems in graph theory having applications in e.g. complexity theory. The method of generating functions is a particularly effective method for a large class of such problems making use of basic results from complex analysis

- UNBOUNDED OPREATORS AND SELF-ADJOINTNESS [An2]
  Many of the interesting operators playing a role in mathematical physics, in particular differential operators of use in classical and quantum mechanics, are unbounded. The extension of fundamental results valid for bounded operators on a Hilbert space, such as the notion of adjoint operator and diagonalisation properties, is therefore of importance and turns out to be non-trivial

Previous projects:

- CLIFFORD ALGEBRAS, SPIN GROUPS AND DIRAC OPERATORS [Alg1,An2]
- RAMSEY THEORY [Dis1,An1]
- CAUSAL STRUCTURES [An1,Geom2]
- THE TUTTE POLYNOMIAL [Dis1,An1]
- KNOT THEORY AND STATISTICAL MECHANICS [Dis1,AN1]
- GRAPH 3-COLOURINGS [Dis1,An1]
- MINIMAL SURFACES [Geom1,An1]
- PLANAR GRAPHS [Dis1,AN1]

4.2 Jan Philip Solovej

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Relevant interests:

Mathematical Physics, Quantum Mechanics, Spectral Theory, Partial Differential Equations

Suggested projects:

- PARTIAL DIFFERENTIAL EQUATIONS: THE WAVE EQUATION AND THE HEAT EQUATION [An1,An2]
  The goal of the project is to derive the solution formulas for the initial value problems for the wave equation (homogeoneous and inhomogeneous) and the heat equation. Moreover, the project will focus on the strong and weak Huygens principles for the wave equation and on the maximum principle for the heat equation.

  Literature: F. John, Partial Differential Equations or L.C. Evans Partial Differential equations.

- FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS [An1,An2]
  The project considers first order partial differential equations, in particular, quasi-linear equations. The goal is to describe the method of characteristics and to study equations that exhibit schock formation.

  Literature: F. John, Partial Differential Equations

4.3 Henrik L. Pedersen

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Relevant interests:

Complex analysis. Special functions. Orthogonal polynomials and moment problems.

Suggested projects:

- THE GAMMA FUNCTION [An1,KomAn]
  Euler's Gamma function is the most import of the non-elementary functions. It gives a continuous version of the numbers $n!$ and enters in all kinds of applications from probability to physics.

- ENTIRE FUNCTIONS [An1, Koman]
  Entire functions are represented by power series with infinite radius of convergence. They can be classified in terms of their growth properties.

- BOUNDARY BEHAVIOUR OF POWER SERIES [KomAn]
  A power series converges inside the disk of convergence, and diverges outside the circle. What happens on the boundary? The sum of the geometric series has a holomorphic extension to the entire complex plane except 1. If we remove a lot of terms from this series it turns out that the sum function has the unit circle as a natural boundary, meaning that it cannot be extended holomorphically to any arc of that circle. What is going on?

- SUBHARMONIC FUNCTIONS [KomAn, Measure theory]
  The real part u of a holomorphic function is harmonic, meaning that its Laplacian is zero: $\Delta u = \partial^2_{xx}u + \partial^2_{yy}u \equiv 0$. A subharmonic function u in the complex plane satisfies $\Delta u \geq 0$. For these functions versions of the maximum principle and of Liouville's theorem hold

- PICARD'S THEOREMS [KomAn, some measure theory]
  If you are presented with an entire function f and you have two different complex numbers not in the image set $f(\mathbb{C})$ then f is constant. This result is known as Picard's little theorem.

- RIEMANN'S MAPPING THEOREM [KomAn]
  Any simply connected region $\mathcal{D}$ in the complex plane except the plane itself is conformally equivalent to the open unit disk Δ, meaning that there exists a holomorphic and bijective mapping $\varphi : \Delta \to \mathcal{D}$.

- CAUCHY'S INTEGRAL THEOREM [KomAn, Topology]
  Cauchy's theorem holds true in simply connected regions. This can be proved by studying the winding number and homotopic curves.

- PALEY–WIENER'S THEOREM [KomAn, some measure theory]
  The Fourier transform $\widehat{\phi}$ of a function ϕ from the Hilbert space $L_2(-a, a)$ can be extended to an entire function of exponential type (meaning that its growth is dominated by $e^{K|z|}$ for all large $|z|$. Conversely, any entire function of enxponential type is in fact the Fourier transform of an L_2-function of a finite interval.

4.4 Morten S. Risager

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Relevant interests:

Number theory, automorphic forms, complex analysis, Riemann surfaces.

Suggested projects:

- THE PRIME NUMBER THEOREM [KomAn, An2]
  The prime number theorem gives a quantitative version of Euclid theorem about the infinitude of primes: it describes how the primes are distributed among the integers. It was conjectured 100 years before the first proof.

- Twin primes and sieve theorems [KomAn, An2]
  Very little is known about the number of twin primes. Using sieve methods one can show that the sum of reciprocicals of twin primes is convergent. Still it is not known if there are only finitely many or not.

- The functional equation for Riemann's zeta function [KomAn, An2]
  Using methods from Fourier analysis - in particular Poisson summation - one investigates the properties of Riemann's famous zeta function.

- Modular forms of $\mathrm{SL}_2(\mathbb{Z})$ [KomAn, Alg1]
  Modular forms on $\mathrm{SL}_2(\mathbb{Z})$ are a special class of highly symmetric analytic functions on the upper halfplane. It turns out that many counting problems in number theory can be encoded into the Fourier coefficients of such modular forms.

- p-adic numbers [Alg1, An1]
  The real numbers can be constructed from the rational numbers $\mathbb{Q}$ as Cauchy sequences with respect to the standard norm on $\mathbb{Q}$. For each prime number there is another norm on $\mathbb{Q}$ called the p-adic norm, and by going through the same construction which lead to $\mathbb{R}$ we arrive at the p-adic number $\mathbb{Q}_p$. This is a beautiful object and is highly useful in number theory.

Previous projects:

- Elementary methods in number theory, and a theorem of Terrence Tao. [An2, ElmTal]
- Primes in arithmetic progressions [KomAn, An2]
- Small eigenvalues of the automorphic Laplacian and Rademachers conjecture for congruence groups [KomAn, An3]
- Weyls law and the Gauss' circle problem [An2]
- The Bombieri-Vinogradov theorem [AnTal]
- Primes close together [AnTal]
- Dirichlets unit theorem [Alg 2]
- Higher order reciprocity [Elmtal, Alg2]
- The Agrawal-Kayak-Saxena algorithm for primality testing [Alg2]
- Pell's equation and Archimedes' revenge [ElmTal, Alg2]
- The phragmén Lindelöf principle [KomAn]

4.5 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- THE HEISENBERG GROUP [An1,An2]
  The Heisenberg group is important, for example because it is generated by the position and momentum operators in quantum mechanics. The purpose of this project is to study its representation theory. A famous theorem of Stone and von Neumann relates all irreducible representations to the Schrödinger representation acting on $L^2(\mathbf{R}^n)$.

- UNCERTAINTY PRINCIPLES [An1,Sand1,KomAn]
  Various mathematical formulations of the Heisenberg uncertainty principle are studied. Expressed mathematically, the principle asserts that a non-zero function f on $\mathbf{R}$ and its Fourier transform $\hat{f}$ cannot be simultaneously concentrated. A precise version, called the Heisenberg inequality, expresses this in terms of standard deviations. A variant of the theorem, due to Hardy, states that f and $\hat{f}$ cannot both decay more rapidly than a Gaussian function.

- THE PETER-WEYL THEOREM [An1,An2,Sand1]
  The purpose of this project is to study $L^2(G)$ for a compact group G, equipped with Haar measure. The theorem of Peter and Weyl describes how this space can be orthogonally decomposed into finite dimensional subspaces, which are invariant under left and right displacements by G. Existence of Haar measure can be proved or assumed.

4.6 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- SOLITONS IN KORTEWEG-DE VRIES' EQUATION [An1]
  Waves normally either spread out or break with time. However, the surprising emergence of special "solitary waves" (a.k.a. "solitons") which retain a fixed profile with time can occur. Historically, this was first observed by Scottish naval engineer John Scott Russell in 1834, in a shallow water canal: This "translating

wave" of water, as he named it, was so stable in shape that he could follow it by horseback long enough to determine its speed to be approximately 8 miles/hour. The KdV equation models this behavior and has many interesting features that you can investigate, such as the collision of several solitons.

- CURVE SHORTENING FLOW AND ISOPERIMETRY [An1,An2,Geom1]
The mean curvature flow for planar curves is used for proving the isoperimetric inequality: Of all the simple closed planar curves enclosing the same area, the round circle has the least length. It is actually essential for the proof that the circle is a geometric soliton, see the KdV project. (Especially the non-convex case is very hard, so do not expect to finish the full proof).

- BROWNIAN MOTION AND THE HEAT EQUATION [An1, Sand1]
One can start with the random walk as a simple model for a diffusion process and derive the heat equation (and understand Einstein's proof from one of his three famous 1905 papers). There are extensions to modern versions involving fractional differentiation operators. I.e. how do you differentiate "half a time"?

5 Geometry

5.1 Henrik Schlichtkrull

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Relevant interests:

Geometry, Lie groups, Analysis, Harmonic analysis, Representation Theory

Suggested projects:

- GLOBAL PROPERTIES OF CURVES (AND/OR SURFACES) [Geom1,An1]
  The differential geometry studied in Geometry 1 is of a local nature. The curvature of a curve in a point, for example, describes a property of the curve just in the vicinity of that point. In this project the focus is on *global* aspects of closed curves, as for example expressed in Fenchel's theorem, which gives a lower bound for the total integral of the curvature, in terms of the perimeter.

- GEODESIC DISTANCE [Geom1,An1]
  The geodesic distance between two points on a surface is the shortest length of a geodesic joining them. It turns the surface into a metric space. The project consists of describing some propreties of the metric. For example Bonnet's theorem: *If the Gaussian curvature is everywhere* ≥ 1, *then all distances are* $\leq \pi$.

5.2 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- MATHEMATICAL SOAP BUBBLES: MINIMAL SURFACES [An1]
  There is a rich theory for surfaces of (zero or positive) constant mean curvature, and many interesting examples to visualize. Important aspects of existence and uniqueness can be studied (the difficulty of the project accordingly adjusted).

- VERY LONG GEODESICS: LIBERMAN'S THEOREM [An1, Geom1]
  The theorem states that on a closed 2-dimensional surface with everywhere positive Gauß curvature, geodesic curve segments cannot be arbitrarily long and still remain injective: They must eventually self-intersect. One can also prove an upper bound for the length.

- The Uniformization Theorem for Surfaces [An1,An2,Geom1/KomAn]
  A special case of this classical theorem states that: Every closed 2-dimensional surface has a Riemannian metric of constant Gauß curvature. (This is a long and complicated proof, so do not expect your thesis to contain it in all details).

5.3 Elisenda Feliu

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Relevant interests:

applied algebraic geometry, computational algebra, mathematical biology, biochemical reaction networks

Suggested projects:

- Topics on polytopes [LinAlg, An1, Geom1, Alg2]
  A polytope is the convex hull of a finite set of points in $\mathbb{R}^n$. This project focuses on studying the basic properties of polytopes and how to effectively compute it given a set of points. A possible textbook is the first part of "Polyhedral and Algebraic Methods in Computational Geometry", by Joswig and Theobald. The project can also include the study of mixed volumes and how the number of solutions of a system of polynomial equations depends on properties of the so-called Newton polytope (the convex hull of the exponents of the monomials).

5.4 Hans Plesner Jakobsen

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Relevant interests:

Unitaritet, Liegrupper, Liealgebraer, kvantiserede matrixalgebraer, kovariante differentialoperatorer i matematisk fysik, kvantiserede indhyldningsalgebraer

Suggested projects:

- Symmetrier [ca. 1 års matematik]
  - Diskrete symmetrier: Tapetgrupper, Krystallografiske grupper.
  - Kontinuerte symmetrier: Rotationsgruppen, Lorentzgruppen, PoincarÈgruppen, den konforme gruppe,...
- Gauss-Bonnet [Forudsætter ca. 1 års matematik]
  Den måske mest fundamentale sætning i Euklidisk geometri er Thales' sætning, der siger, at summen af vinklerne i en trekant er 180°. Denne sætning kan generaliseres til trekanter påglatte flader i rummet. Herved fremkommer Gauss-Bonnet's sætning i lokal udgave. Den globale udgave af sætningen leder til en invariant for kompakte flader: Eulerkarakteristikken.

- De kanoniske kommutatorrelationer [ca. 1 års matematik + Hilbertrum]
  Operatorerne Q og P givet ved $(QF)(x) = xF(x)$ of $(PF)(x) = -i(\frac{dF}{dx})(x)$ er forbundet via Fouriertransformationen, men kan Fouriertransformationen 'konstrueres' ud fra disse? Kan man bygge en bølgeoperator eller en Diracoperator ud af den harmoniske oscillator?

- Liealgebraer [ca. 1 års matematik + (kan aftales)]
  (F.eks.) Klassifikation. Dynkin diagrammer, Kac-Moody algebraer, super Liealgebraer. Hvad fik Borcherds (bl.a.) Fieldsmedaljen for?

- Matrix Liegrupper [ca. 1 års matematik]
  Bl.a. eksponentialfunktionen for matricer, tensorprodukter, duale vektorrum. Er der en forbindelse mellem Peter Weyl Sætningen og Stone Weirstrass Sætningen?

5.5 Other projects

Other projects in this area can be found with

- Nathalie Wahl (7.3)

6 Noncommutativity

You can find links to past projects within this field on

`math.ku.dk/english/research/tfa/ncg/paststudents/`

6.1 Søren Eilers

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Relevant interests:

Advanced linear algebra related to operator algebras. Enumerative combinatorics. Dynamical systems. Mathematics in computer science; computer science in mathematics.

Suggested projects:

- Experimental mathematics [XM]
  Design a large-scale computer experiment to investigate a mathematical problem and draw mathematical conclusions from it. **Note:** Only offered to students who have taken the XM course (or take it concurrently with the project).
- Perron-Frobenius theory with applications [LinAlg, An1]
  Methods involving matrix algebra lead to applications such as Google's PageRank and to the ranking of American football teams.
- Combinatorial game theory [Dis2]
  Learn the basics of combinatorial game theory from the classical text by Conway, and analyze one or more hitherto unstudied games.
- Data storage with symbolic dynamics [An1, Dis]
  Engineering constraints neccessitate a recoding of arbitary binary sequences into sequences meeting certain constraints such as "between two consecutive ones are at least 1, and at most 3, zeroes". Understanding how this is done requires a combination of analysis and discrete mathematics involving notions such as entropy and encoder graphs.

Previous projects:

- Combinatorial game theory [Dis2]
- An experimental approach to flow equivalence [An1, XM]
- An experimental approach to packing problems [XM]
- Fatou- and Julia-sets [KomAn]
- Growth constants for pyramidal structures [XM]

- Lehmer's totient problem [Dis2]
- Liapounov's theorem [MI]
- Planar geometry in high school mathematics [Geom1]
- Visualization of non-euclidean geometry [Geom1]

6.2 Niels Grønbæk

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Relevant interests:

Banachrum, banachalgebra, kohomologi, matematikkens didaktik

Suggested projects:

- Et undervisningsforløb på gymnasialt niveau [LinAlg, An1, Alg1, Geo1]
  Projektet går ud på at tilrettelægge, udføre og evaluere et undervisningsforløb af ca. 2 ugers varighed i en gymnasieklasse.
- Amenable Banach Algebras [An3]
  Amenability of Banach algebras is an important concept which originates in harmonic analysis of locally compact groups. In the project you will establish this connection and apply it to specific Banach algebras such as the Banach algebra of compact operators on a Hilbert space.

6.3 Magdalena Musat

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Relevant interests:

Banach Spaces, Functional Analysis, Operator Algebras, Probability Theory

Suggested projects:

- Geometry of Banach spaces [Analysis 3]
  A number of very interesting problems concerning the geometry of Banach spaces can be addressed in a bachelor project. For example, does every infinite dimensional Banach space contain an infinite dimensional reflexive subspace or an isomorphic copy of l_1 or c_0? Or, does there exist a reflexive Banach space in which neither an l_p-space, nor a c_0-space can embed? Another project could explore the theory of type and cotype, which provides a scale for measuring how close a given Banach space is to being a Hilbert space.

- Convexity in Banach spaces [Analysis 3]
  The question of differentiability of the norm of a given Banach space is closely related to certain convexity properties of it, such as uniform convexity, smoothness and uniform smoothness. This project will explore these connections, and study further properties of uniformly convex (respectively, uniformly smooth) spaces. The Lebesgue spaces L_p $(1 < p < \infty)$ are both uniformly convex and uniformly smooth.

- Haar measure [MI]
  This project is devoted to the proof of existence and uniqueness of left (respectively, right) Haar measure on a locally compact topological group G. For example, Lebesgue measure is a (left and right) Haar measure on $\mathbb{R}$, and counting measure is a (left and right) Haar measure on the integers (or any group with the discrete topology).

- Fernique's theorem [SAND 1, Analysis 3]
  This project deals with probability theory concepts in the setting of Banach spaces, that is, random variables taking values in a (possibly infinite dimensional) Banach space. Fernique's theorem generalizes the result that gaussian distributions on $\mathbb{R}$ have exponential tails to the (infinite dimensional) setting of gaussian measures on arbitrary Banach spaces.

6.4 Ryszard Nest

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Relevant interests:

Non-Commutative Geometry, Deformation Theory, Poisson Geometry

Suggested projects:

- Clifford Algebras [LinAlg, Geom 1]
  Clifford algebra is a family $\mathcal{C}^{p,q}$ of finite dimensional algebras associated to non-degenerate bilinear forms which play very important role in both topology and geometry. The simplest examples are $\mathbb{R}$, $\mathbb{C}$ and the quaternion algebra $\mathbb{H}$. The main result is the periodicity modulo eight of $\mathcal{C}^{p,q}$, which has far reaching consequences (e.g., Bott periodicity, construction of Dirac operators) in various areas of mathematics.

- Axiom of choice and the Banach-Tarski paradox [LinAlg, Analysis 1]
  The axiom of choice, stating that for every set of mutually disjoint nonempty sets there exists a set that has exactly one member common with each of these sets, is one of the more "obvious" assumptions of set theory, but has far reaching consequences. Most of modern mathematics is based on its more or less tacit assumption. The goal of this project is to study equivalent formulations of the

axiom of choice and some of its more exotic consequences, like the *Banach-Tarski paradox*, which says that one can decompose a solid ball of radius one into five pieces, and then rearrange those into two solid balls, both with radius one.

- FORMAL DEFORMATIONS OF $\mathbb{R}^{2n}$ [LinAlg, Geom 1]
  The uncertainty principle in quantum mechanics says that the coordinate and momentum variables satisfy the relation $[p, x] = \hbar$, where $\hbar$ is the Planck constant. This particular project is about constructing associative products in $C^\infty(\mathbb{R}^{2n})[[\hbar]]$ satisfying this relation and studying their properties.

6.5 Mikael Rørdam

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Relevant interests:

Operator Algebras, Topics in Measure Theory, Discrete Mathematics

Suggested projects:

- TOPICS IN C^*-ALGEBRAS [Analysis 3]
  C^*-algebras can be defined either abstractly, as a Banach algebra with an involution, or concretely, as subalgebras of the algebra of bounded operators on a Hilbert space. They can be viewed as non-commutative analogues of spaces, since every commutative C^*-algebra is equal to the set of continuous functions on a locally compact Hausdorff space. Several topics concerning C^*-algebras and concerning the study of specific examples of C^*-algebras, can serve as interesting topics for a bachelor project.

- TOPICS IN MEASURE THEORY [MI]
  We can here look at more advanced topics from measure theory, that are not covered in MI, such as existence (and uniqueness) of Lebesgue measure, or more generally of Haar measure on locally compact groups. Results on non-measurability are intriguing, perhaps most spectacularly seen in the Banach-Tarski paradox that gives a recipe for making two solid balls of radius one out of a single solid ball of radius one!

- TOPICS IN DISCRETE MATHEMATICS [Dis2 & Graf]
  One can for example study theorems about coloring of graphs. One can even combine graph theory and functional analysis and study C^*-algebras arising from graphs and the interplay between the two (in which case more prerequisites are needed).

Previous projects:

- IRRATIONAL AND RATIONAL ROTATION C^*-ALGEBRAS [Analyse 3]

- CONVEXITY IN FUNCTIONAL ANALYSIS [Analyse 3]

- The Banach-Tarski Paradox [MI recommended]

7 Topology

You can find links to past projects within this field on

`math.ku.dk/english/research/tfa/top/paststudents/`

7.1 Jesper Grodal

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Relevant interests:

Topology, Algebra, Geometry.

Suggested projects:

- GROUP COHOMOLOGY [Alg2]
  To a group G we can associate a collection of abelian groups $H^n(G)$, $n \in \mathbf{N}$, containing structural information about the group we started with. The aim of the project would be to define these groups, examine some of their properties, and/or examine applications to algebra, topology, or number theory. See e.g.: K.S. Brown: Cohomology of groups

- GROUP ACTIONS [Top, Alg2]
  How can groups act on different combinatorial or geometric objects? Eg. which groups can act freely on a tree? See e.g.: J.-P. Serre: Trees.

- THE BURNSIDE RING [Alg2]
  Given a group G we can consider the set of isomorphism classes of finite G-sets. These can be "added" and "multiplied" via disjoint union and cartesian projects. By formally introducing additive inverses we get a ring called the Burnside ring. What's the structure of this ring and what does it have to do with the group we started with? See:
  `http://en.wikipedia.org/wiki/Burnside_ring`

- THE CLASSIFICATION OF FINITE SIMPLE GROUPS [Alg2]
  One of the most celebrated theorems in 20th century mathematics gives a complete catalogue of finite simple groups. They either belong to one of three infinite families (cyclic, alternating, or classical) or are one of 26 sporadic cases. The aim of the project is to explore this theorem and perhaps one or more of the sporadic simple groups. See:
  `http://en.wikipedia.org/wiki/Classification_of_finite_simple_groups`

- THE PLATONIC SOLIDS AND THEIR SYMMETRIES [Top, Alg2]
  A Platonic solid is a convex polyhedron whose faces are congruent regular polygons, with the same number of faces meeting each vertex. The ancient greeks already knew that there were only 5 platonic solids. The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. The aim

of the project is to understand the mathematics behind this. See: `http://en.wikipedia.org/wiki/Platonic_solid`

- TOPOLOGICAL SPACES FROM CATEGORIES [Top, Alg2]
  Various algebraic or combinatorial structures can be encoded via geometric objects. These "classifying spaces" can then be studied via geometric methods. The goal of the project would be to study one of the many instances of these this, and the project can be tilted in either topological, categorical, or combinatorial directions. See e.g.: A. Bj?rner, Topological methods. Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.

- SIMPLICIAL COMPLEXES IN ALGEBRA AND TOPOLOGY [Alg1, Top]
  The goal of this project is to understand how simplicial complexes can be used to set up a mirror between notions in topology and algebra. For instance, the algebraic mirror image of a topological sphere is a Gorenstein ring.

Previous projects:

- STEENROD OPERATIONS—CONSTRUCTION AND APPLICATIONS [AlgTopII]
- HOMOTOPY THEORY OF TOPOLOGICAL SPACES AND SIMPLICIAL SETS [AlgTopII]
- AUTOMORPHISMS OF G - WITH APPLICATIONS TO GROUP EXTENSIONS [AlgTopII, CatTop]

7.2 Jesper Michael Møller

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Relevant interests:

All kinds of mathematics.

Suggested projects:

- POINCARÉ SPHERE [Topology, group theory]
  What are the properties of the Poincaré sphere?

- TOPOLOGICAL COMBINATORICS [Dis1, Top]
  Combinatorial problems, such as determining chromatic numbers of graphs, can be solved using topological methods.

- PARTIALLY ORDERED SETS [Dis1]
  Partially ordered sets are fundamental mathematical structures that lie behind phenomena such as the Principle of Inclusion-Exclusion and the Möbius inversion formula.

- CHAOS [General topology]
  What is chaos and where does it occur?

- Project of the day [Mathematics]
  http://www.math.ku.dk/~moller/undervisning/fagprojekter.html

7.3 Nathalie Wahl

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Relevant interests:

Graphs, surfaces, manifolds, knots, algebraic structures.

Suggested projects:

- Knots [Alg1,Top]
  Mathematically, knots are embeddings of circles in 3-dimensional space. They are rather complicated objects that can be studied combinatorially or via 3-manifolds. The project consists of learning some basics in knot theory. See for example http://www.earlham.edu/~peters/knotlink.htm.

- Braid groups, configuration spaces and links [Alg1,Top]
  The braid group on n strands can be defined in terms of braids (or strings), or as the fundamental group of the space of configurations of n points in the plane. It is related to knots and links, and also to surfaces. The project consists of exploring braid groups or related groups like mapping class groups. See for example J. Birman, Braids, links, and mapping class groups.

- Classification of surfaces [Top,Geom1]
  Closed 2-dimensional surfaces can be completely classified by their genus (number of holes). There are several ways of proving this fact and the project is to study one of the proofs. See for example W. Massey, A Basic Course in Algebraic Topology, or A. Gramain, Topology of Surfaces.

- 3-manifolds [Top,Geom1]
  3-dimensional manifolds are a lot harder to study than 2-dimensional ones. The geometrization conjecture (proved recently by Perelman) gives a description of the basic building blocks of 3-manifolds. Other approaches to 3-manifolds include knots, or "heegaard splittings", named after the Danish mathematician Poul Heegaard. The project consists of exploring the world of 3-manifolds. See for example http://en.wikipedia.org/wiki/3-manifolds.

- Non-Euclidean geometries [Geom1]
  Euclidean geometry is the geometry we are used to, where parallel lines exist and never meet, where the sum of the angles in a triangle is always 180°. But there are geometries where these facts are no longer true. Important examples are the hyperbolic and the spherical geometries. The project consists of exploring non-euclidian geometries. See for example
  http://en.wikipedia.org/wiki/Non-euclidean_geometries

- Frobenius algebras, Hopf algebras [LinAlg,Alg1]
  A Frobenius algebra is an algebra with extra structure that can be described algebraically or using surfaces. A Hopf algebra is a similar structure. Both types of algebraic structures occur many places in mathematics. The project consists of looking at examples and properties of these algebraic structures. See for example J. Kock, Frobenius algebras and 2D topological quantum field theories.
- Khovanov Homology [AlgTop – or familiarity with category theory]
  The complexity of knots is immense. Explore `http://katlas.org/`. Over the last 100 years various tools have been developed to distinguish and classify knots. A lot of work is still needed to have a good understanding of the world of knots.

  This project would aim at understanding one of the stronger tools available to this date; Khovanov Homology.
- Operads and Algebras [Alg2]
  Operads is an effective tool to cope with exotic algebraic structures. How do you for instance work with algebraic structures that are not (strictly) associative? Depending on interest, the project can have a more algebraic or more topological flavor.
- Morse Theory [Geom2 – for instance simultaneously]
  The second derivative test, known from MatIntro, tells you about local behavious of a 2-variable function. Expanding this test to manifolds in general yields Morse Theory, which plays a key role in modern geometry.

  This project would start out by introducing Morse Theory. Various structure and classification results about manifolds could be shown as applications of the theory.

8 History and philosophy of mathematics

8.1 Jesper Lützen

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Relevant interests:

History of Mathematics

Suggested projects:

- The history of non-Euclidean geometry [Hist1, preferrably VtMat]
  How did non-Euclidean geometry arise and how was its consistency "proved". How did the new geometry affect the epistemology of mathematics?
- The development of the function concept [Hist1]
  How did the concept of function become the central one in mathematical analysis and how did the meaning of the term change over time.
- Archimedes and his mathematics [Hist1]
  Give a critical account of the exciting life of this first rate mathematician and analyze his "indivisible" method and his use of the exhaustion method.
- What is a mathematical proof, and what is its purpose [Hist1, VtMat]
  Give philosophical and historical accounts of the role(s) played by proofs in the development of mathematics

Previous projects:

- A brief history of complex numbers [Hist1, preferrably KomAn]
- Mathematical induction. A history [Hist1]
- Aspects of Euler's number theory [Hist1, ElmTal]
- Mathematics in Plato's dialogues [Hist1, VtMat]
- Axiomatization of geometry from Euclid to Hilbert [Hist1, preferrably VtMat]
- Lakatos' philosophy applied to the four color theorem [Dis, Hist1]
- History of mathematics in mathematics teaching: How and why [Hist1, DidG preferrably DidMat]

8.2 Henrik Kragh Sørensen og Mikkel Willum Johansen

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Relevant interests:

Matematikkens videnskabsteori, matematikkens historie.

Suggested projects:

- DIT EGET PROJEKT? []
  I videnskabsteori vil vi typisk i et samarbejde mellem den studerende og underviseren søge at designe individuelle projekter, der passer til den enkelte studerendes faglige profil og interesser. Som nogle eksempler på mulige projektemner kan vi dog nævne de nedenstående:

- COMPUTERASSISTEREDE BEVISER [VtMat]
  Computere spiller en stadig stigende rolle i matematisk forskningspraksis. Igennem at studere eksempler på computer-assisterede beviser (fra Firefarvesætningen) og fremad kan man udfordre nogle af vores filosofiske antagelser om matematik. Man har fx diskuteret, hvorvidt matematisk viden, som baserer sig på computerbeviser, stadig er a priori viden.

- ALGEBRA I 1800-TALLET [Hist1, VtMat]
  I løbet af 1800-tallet blev algebraen udviklet fra en teori om ligningsløsning til en generel teori om algebraiske strukturer. Igennem et nedslag i fx Gauss' studier af hvornår cirkelperiferien kan inddeles i n lige store stykker med passer og lineal (1801), Abels bevis for femtegradsligningens algebraiske uløselighed (1826), eller gruppebegrebets aksiomatisering i anden halvdel af 1800-tallet kan vi historisk og videnskabsteoretisk diskutere matematikkens udvikling.

- KILDECENTRERET MATEMATIKHISTORIE I GYMNASIET [VtMat, Hist1, DidMat]
  Hvordan kan man bruge autentiske matematiske kilder i gymnasiets matematikundervisning og til tværfaglig undervisning? Hvordan skal kilden forberedes? Hvilke undervisningsmål kan sådan en tilgang være med til at opfylde?

- ØKONOMISKE MODELLER []
  Diskuter muligheden for at opstille værdifri modeller i økonomien. Diskuter resultatet fx i relation til værdi-fakta-skellet i samfundsdebatten.

  Litteraturforslag: Hausman & McPherson: *Economic analysis, moral philosophy, & public policy*; Klemens Kappel: *Videnskabens særlige rolle i det liberale demokrati*

- KOGNITIV SEMANTIK []
  Beskriv G. Lakoff & R. Núñez' forsøg på at give matematikken et grundlag i menneskets biologi og kognition. Diskuter rimeligheden af dette grundlag fx

ved at inddrage og undersøge cases eller eksempler taget fra dine andre matematikkurser.

Litteraturforslag: Lakoff & Núñez: *Where mathematics comes from.*

- Symboler og diagrammer i matematikken []
  Den moderne matematiske praksis er intimt sammenkædet med brugen af højt specialiserede symboler og diagrammer. Beskriv en eller flere filosofiske teorier om brugen af den type repræsentationer. Det kan fx være teorier som C.S. Perices teori om diagrammatisk tænkning eller den mere moderne teori om distribueret kognition. Analyser med udgangspunkt i den valgte teori et eller flere eksempler på brug af diagrammer og symboler taget fra dine øvrige kurser. Vurder og diskuter på den baggrund validiteten af den valgte teori.

  Litteraturforslag: Hutchins: *Material anchors for conceptual blends*; Kirsh: *Thinking with external representations*; De Cruz: *Mathematical symbols as epistemic actions – an extended mind perspective*; Peirce: *Polegomena for an apology to pragmatism.*

- Alain Connes' platonisme []
  Den franske matematiker Alain Connes er et af de bedste eksempler på en moderne platonisk i matematikken. Beskriv Alain Connes' bud på en moderne platonisme og diskuter med inddragelse af relevant filosofisk teori i hvilken grad projektet lykkes.

  Litteraturforslag: Changeux & Connes: *Conversations on mind, matter, and mathematics.*

- Økonomiske modellers performativitet []
  Diskuter, fx med udgangspunkt i Black-Scholes-modellen i hvilken grad og gennem hvilke mekanismer matematiske modeller kan påvirke og ændre samfundet.

  Litteraturforslag: MacKenzie: *An Equation and its Worlds: Bricolage, Ecemplars, Disunity and Performativity in Financial Economics*; Callon: *The laws of the markets.*

- Domæneudvidelse i matematikken []
  Gennem en analyse af Hamiltons udvidelse af komplekse tal til kvaternionerne opstiller Andrew Pickering en mekanisme, der skal forklare hvordan domæneudvidelser generelt foregår i matematikken. Beskriv Pickerings mekanisme og diskuter hvor rimelig den er på baggrund af analyser af lignende cases.

  Litteraturforslag: Pickering: *The mangle of practice.*

9 Teaching and Didactics of Mathematics

9.1 Carl Winsløw

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Relevant interests:

Didactics of Mathematics

Suggested projects:

- Mathematical content analysis and design [One or more B.Sc.-courses in mathematics ; DidG]

  You select a topic (result, concept, object,...) from secondary mathematics, about which you have acquired significant new knowledge during your bachelor studies in mathematics. Your paper should then include two parts:

  1. A concise exposition on the topic, based on central results and methods from one or more courses from the B.Sc. programme in mathematics (excl. MatIntro).
  2. Design and theoretical analysis of a didactic situation for this topic (at secondary level).

  Here are some example of suitable topics:

  - angles
  - exponential functions
  - linear regression
  - $\mathbb{R}$

  It is of course much better if you have another and more exciting idea yourself.

9.2 Other projects

Other projects in this area can be found with

- Niels Grønbæk (6.2)
- Jesper Lützen (8.1)
- Henrik Kragh Sørensen og Mikkel Willum Johansen (8.2)

10 Set Theory

10.1 Asger Tornquist

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Relevant interests:

Mathematical logic, descriptive set theory, recursion theory, descriptive combinatorics, ergodic theory.

Suggested projects:

- Recursion theory []
  Recursion theory, also called computability theory, is concerned with the study of computable functions on the natural numbers. There are many ways of defining computable functions, the most famous being via so-called Turing machines.

  A project in this area could focus on building up the elementary tools of recursion theory, and then move on to more advanced topics such as relative computability, recursively enumerable degrees, or classical decision problems.

- Descriptive set theory and ergodic theory []
  Descriptive set theory is the study of the definable sets and functions that could (and do) arise in analysis. The most tangible class of such sets and functions are the Borel sets and functions, known from measure theory.

  On the other hand, ergodic theory is the study of measure-preserving group actions on measure spaces, and has its origin in statistical mechanics in physics.

  A lot of work in descriptive set theory these days focusses on groups, their actions, and their "orbit equivalence relations", and this has produced a growing overlap of interested with ergodic theory. A project in this area could focus on ergodic theoretic results such as Dye's theorem and orbit equivalence rigidity, or it could focus on descriptive set theoretic dichotomies, and "orbit cardinals" associated to groups actions.

- Ramsey theory, finite and infinite []
  Ramsey theory is a theory of to high-dimensional pigeonhole principles. One way of stating the classical pigeonhole principle is to say: If $n > m$ objects are coloured in m colours, there must be two objects that get the same colour. Ramsey theorists say that there is a "monochromatic" set of size at least 2.

  Somewhat surprisingly, there are higher dimensional versions of this: For instance, if 6 random people walk into a room, then there are 3 people who either all knew each other in advance, or there are 3 people who didn't know each other in advance. That's a special case of the classical Ramsey theorem.

  Ramsey theory, a major area of modern combinatorics, is about the various higher-dimensional pigeonhole principles, as well as their infinitary analogues. A project in this area could start with the classical finite Ramsey theorem, and other finite analogues, and then perhaps move into the more challenging setting of infinitary Ramsey theory, which uses ideas from descriptive set theory and topology.

Previous projects:

- Recursion theory []
- Banach-Tarski's paradox with Baire measurable pieces []
- Hjorth's turbulence theory []
- Ergodic theory and Dye's theorem []
- Gödel's constructible universe L []
- Infinitary combinatorics in L []
- Martin's axiom and applications []

11 Applied mathematics

11.1 Elisenda Feliu

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Relevant interests:

applied algebraic geometry, computational algebra, mathematical biology, biochemical reaction networks

Suggested projects:

- QUALITATIVE ANALYSIS OF ORDINARY DIFFERENTIAL EQUATIONS [LinAlg, An1]
  This project focuses on the study of qualitative properties of systems of differential equations, mainly focusing on steady states, like the determination of stability and bifurcations. The project can be developed from a purely theoretical perspective, or combined with a hands-on analysis of specific systems appearing in real world scenarios.

  Literature: book "Differential equations, dynamical systems, and an introduction to chaos", by Hirsch, Smale, Devaney.

- THE MATHEMATICS OF BIOCHEMICAL REACTION NETWORKS [LinAlg, An1]
  In molecular biology, biochemical reaction networks are studied by means of a system of ordinary differential equations. These systems have a particular form, from which dynamical properties can be studied. The goal of the project is to learn the mathematical framework underlying the study of reaction networks, and afterwards study a specific system of interest.

12 Probability

12.1 Ernst Hansen

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Relevant interests:

Probability, analysis, statistics

Suggested projects:

- Geometric measure theory [MI]
  There are a number of interesting results on the boundary between measure theory and geometry: the coarea formula on decomposition of integrals (polar integration decomposes a planar integral as integrals over circles - the coarea formula generalises this to a variety of other decompositions), Sard's theorem on the measure of critical images, Stoke's theorem (allows us to replace the integral over at set A with an integral over ∂A, thus generalising the fundamental theorem of calculues). A project could focus on smooth versions of some of these theorems, based on the notion of a Lebesgue measure on smooth subsets of $\mathbb{R}^k$. Or it could focus on more general versions of the theorems, based on the notion of a Hausdorff measure on a arbitrary subset of $\mathbb{R}^k$.

- Refinements of CLT [Stok2]
  The central limit theorem is a cornerstone in probability theory. It says that properly normalized sums of independent random variables converge weakly to a Gaussian limit. A number of important results elaborate on this convergence: Berry-Esseen's theorem gives explicit bounds for the convergence, Edgeworth expansions and saddlepoint-approximations improve on the convergence (sometimes quite dramatically) by modifying the Gaussian limit with small adjustments. Most of these results are based on a very careful analysis of the characteristic function.

- Poisson approximations [MI]
  Sums of Bernoulli variables are often remarkably well approximated by Poisson distributions. If the Bernoulli variables are iid, the sum has a binomial distribution, and it is easy to see that binomial and Poisson distributions are close. But Poisson approximations can be shown to be valid in a vast variety of non-iid situations as well.

- Isoperimetric inequalities [MI]
  The classical isoperimetric inequality states that among the closed planar curves with a specified length, a circle will maximize the area of the enclosed domain. This was 'known' since the ancient greeks, but a rigorous proof was not obtained until the late 19th century. Today, quite accessible proofs can be given using measure theory. A number of generalizations are possible, for instance to spheres (instead of the plane). Or the project could investigate Gaussian isoperimetric

inequalities, where 'length' and 'area' with respect to Lebesgue measure are replaces by similar notions for Gaussian measures. The Gaussian isoperimetric inequality are the gateway to probability theory in Banach spaces.

- ROBUST STATISTICS [Stat2]
  It can be shown in considerable generality that the asymptotically best estimators are obtained by maximing the likelihood function. But this requires that the model is specified correctly. Maximizing the wrong likelihood can lead to quite disastrous results, even if the models is only slightly wrong. For instance, the MLE is in general very sensitive to the presence of 'outliers', which can be thought of as observations that do not conform to the model. Robust statistics are methods that try to safe-guard against minor deviations from the model.

12.2 Carsten Wiuf

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Relevant interests:

Probability and measure theory, and its applications

Suggested projects:

- EXISTENCE OF MEASURES [MI]
  In extension of topics covered in the course Mål- og Integralteori, one could study the existence of measures, in particular the existence of the Lebesgue measure in $\mathbb{R}^d$.

- CENTRAL LIMIT THEOREM [MI]
  In extension of topics covered in the course Mål- og Integralteori, one could study versions and proofs of the central limit theorem, for example, based on characteristic functions.

- WRIGHT-FISHER MODEL [SS, MI, ideelt Stok]
  A central model in mathematical genetics is the Wright-Fisher model that describes the evolution of a population of N individuals, using Markov chain theory. The project will use Markov chain theory to study properties of the Wright-Fisher model and address questions like "how many generations back in time before you shared an ancestor with your neighbour?" and "Are you genetically related to all your great-great grand parents?". You don't need to know any biology.

13 Statistics

13.1 Niels Richard Hansen

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Relevant interests:

Statistics, probability theory

Suggested projects:

- A PROJECT IN STATISTICS [Stat 1, Stat 2]
  A selection of projects will be offered that are all based on real data and offer the opportunity to do a practical project on applied statistics as well as learning new statistical methodology. Contact me for further information.

14 Mathematics of Quantum Theory

14.1 Matthias Christandl

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Relevant interests:

Quantum Information Theory and Quantum Computation

Suggested projects:

- Bell inequalities [QIT1]
  Quantum mechanics violates Bell´s inequalities, which are inequalities that classical local structures satisfy. When the violation is very high, a stronger notion, known as *rigidity* can appear, where it is possible to infer the structure of measurements and states that have been used to infer the violation. The project will investigate rigidity, taking its starting point in a famous theorem by Tsirelson.
- Quantum Entropy [QIT2]
  Quantum entropy or von Neumann entropy is a concave function of the quantum state. Interestingly, there are more concavity-like properties that quantum entropy satisfies. It is the aim of the project to study these properties.
- Quantum Tomography [QIT3]
  Quantum tomography is the art of inferring a quantum state from measured data. To a large extend, quantum tomography is a problem from classical statistics and it is the goal to collect classical methods to treat this problem.

14.2 Jan Philip Solovej

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Relevant interests:

Mathematical Physics, Quantum Mechanics, Spectral Theory, Partial Differential Equations

Suggested projects:

- Why you cannot hear the shape of a drum [An1,An2]
  For any compact open set in $\mathbb{R}^n$ we can define the discrete set of eigenvalues of the Dirichlet Laplacian. In the 2-dimensional case these are the frequencies you would hear if the domain is played as a drum. Translating, rotating or reflecting a domain does not change the eigenvalues. But is this the only way domains can have the same frequencies? I.e., can you hear the shape of a drum? The answer is no and the project constructs pairs of domains in $\mathbb{R}^2$ that have the same Dirichlet Laplace eigenvalues, but are not isometric. As part of the project

the existence of the Dirichlet Laplace eigenvalues will be constructed. A certain regularity property of eigenfunctions will have to be used, but not proved.

Literature: Notes and exercises and the article Buser, Conway, Doyle, and Semmler,Some planar isospectral domains, IMRN, 1994 (9)

- Poisson summation formula and Gauss circle problem [An1]
The aim is to prove the Posson summation formula using the theory of Fourier series. The Posson summation formula can be used to estimate how close Riemann sums are to Riemann integrals. This will be generalized to functions of several variables and applied to count the integer points in a large ball (Gauss circle problem). The problem is of interest in number theory and in quantum mechanics, where it may be interpreted as the number of states in a Fermi gas below the Fermi level.

- Estimating eigenvalues of Schrödinger operators [An1,An2]
The goal of this project is to estimate the number and the sum of negative eigenvalues for Schrödinger operators. In quantum mechanics the negative eigenvalues of Schrödinger operators represent bound states of quantum systems and their sum represents the total energy of the bound states. This project requires first to introduce the Fourier transform

14.3 Niels Martin Møller

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Relevant interests:

Analysis: PDEs, spectral theory. Geometry: Riemannian geometry, curvature flows. Mathematical physics: Quantum (field) theory, general relativity.

Suggested projects:

- Quantum Field Theory in 1-d: The Casimir Effect [An1, An2, KomAn]
As you may have heard, in a certain sense $1 + 2 + 3 + 4 + \ldots " = " - 1/12$. Properly framed, this even has a direct physical relevance and can be measured in the laboratory as the attractive force (in appropriate units) between two conductors in a vacuum, which is known as the Casimir effect. It also makes sense mathematically because it turns out that there is a canonical way of regularizing the sum to do away with the infinities. One popular approach is via spectral zeta functions for Schrödinger operators, which includes Riemann's zeta function $\zeta(s)$. You can in this project look at simple 1-dimensional "toy" quantum field theories, study different regularization methods and among other things show that the value "$-1/12$" is well-defined.

15 Other areas

15.1 Discrete mathematics

Projects in this area can be found with

- Bergfinnur Durhuus (4.1)
- Søren Eilers (6.1)
- Mikael Rørdam (6.5)

15.2 Aspects of computer science

Projects in this area can be found with

- Søren Eilers (6.1)